TCAR and MCAR Options With Galileo and GPS

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BIOGRAPHIES

Wolfgang Werner received a diploma in Computer Science from the University of Technology in Munich in 1994. He worked as a research associate at the Institute of Geodesy and Navigation (IfEN) in the field of high-precision differential GPS (DGPS), ambiguity resolution and airport pseudolite (APL) research. In 2000 he received his Ph. D. from the University FAF Munich. Since 1999 he is Technical Director of IfEN GmbH. Having been responsible for the EGNOS Independent Check Set algorithm development, he is currently working on Galileo integrity algorithms.

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ABSTRACT

The big number of available Galileo and GPS-IIF signals and frequencies available at the end of this decade makes the use of three-carrier ambiguity resolution (TCAR) or multiple carrier ambiguity resolution (MCAR) approaches interesting. However, not all possibilities perform equally. Due to the different wavelengths that are involved different success rates for a correct fixing of any type of wide-lane ambiguity can be given. All relevant different options of signal combinations have been analysed systematically with respect to their success rates and are presented in the paper. The current Galileo baseline signal structures and the modernized GPS-IIF signal structures have been investigated.

The important point in selecting a well performing signal and frequency combination for TCAR is that in the cascading steps, the noise of the measurement of one step is low with respect to the virtual wavelength of the next steps ambiguity. Having this in mind and having available a certain number of frequencies, this leads to the MCAR approach, where the TCAR steps can be sequentially ordered according to their virtual wavelength to maximise the success rate for correct ambiguity fixing. In literature, the term "gap-bridging" concept is used for the steps of TCAR, where the gap between code pseudorange and finally the base carrier phase measurement accuracy is bridged. Analogously, the MCAR approach presented here makes use of a concept that can be called "fine-gap-bridging" concept. By making use of all available signals the risk for a wrong fixing in any of the steps is minimised.

Furthermore, comparisons to the state-of-the-art ambiguity resolution approaches - sometimes called real-time kinematic (RTK) - have been made. With more than two frequencies available both approaches yield very good results. However, one major difference between both approaches is that in the TCAR (or MCAR) case no geometry information is exploited. This leads to the severe disadvantage that there might be wrong ambiguity fixings. As these wrong fixes can even happen in the early steps of TCAR or MCAR, huge range errors result. If this is the case, then the remaining steps generate a random lower-level range error as a result. Of course, there are several possibilities to perform post-TCAR plausibility checks, but there is no guarantee that could lead to a strong statement about integrity.

This paper reviews the TCAR approach, lists all assumptions that have been made and presents the results of the TCAR/MCAR analyses that have been made. Comparisons to RTK approaches are also discussed. Finally, recommendations based on these

results are given. The main results can be summarised as follows:

1. The code accuracy of Galileo AltBOC (15,120) on E5ab does allow for a (very) safe correct fixing of any super-widelane ambiguity.
2. The signal combinations that do not make use of the E5ab code range for the first step, have a high success rate for the remaining two fixing steps (widelane and base frequency ambiguity fixes).
3. The combination with E5ab, E6-E5ab, L1-E5ab, E5ab seems to be the best under the used assumptions.
4. Compared to GPS TCAR, in Galileo there are combinations with higher success rates, based on the relationships of the available signal frequencies.
5. Using MCAR, the risk of wrong ambiguity fixings is below the 10^{-5} level under the assumptions that have been made here.

CARRIER-PHASE TECHNIQUES

To achieve very high position accuracies (position errors less than about 1 m) the satellite navigation system user needs to exploit not only code pseudoranges but also – and more importantly - the carrier phase measurements. However, the carrier phase measurements are ambiguous with respect to the integer number of cycles from satellite to receiver.

There are several well-known approaches for making use of these high-precision but ambiguous measurements. In the following the methods for RTK (float and fixed) as well as the TCAR/MCAR methods are considered in detail.

Before, however, a clear definition of what is understood under these acronyms is given:

- **Float RTK:** A standard kinematic DGPS solution including carrier-phase measurements. The unknown integer ambiguities are part of the filter state and will be estimated within the positioning (Kalman) filter. They will be used as float values and are not fixed to integers.

- **Fixed RTK:** This approach is the same as in the float RTK case. But, when enough information in the positioning filter is available (variances in the ambiguity states are low or other statistical tests are positive), the ambiguities are fixed to integer values. In the further processing only the fixed values of the ambiguities are used for positioning. The fixing of the ambiguities is done via state-of-the-art algorithms like Euler/Landau search (*Euler and Landau, 1992)* and possibly a pre-search decorrelation (e.g. LAMBDA transformation, see *Teunissen at al., 1995*). It is important to note here, that the ambiguity fixing is performed simultaneously on all ambiguity states based on the covariance matrix of the positioning filter including geometrical information.

- **TCAR:** This is a quite new approach proposed by *Harris (1997)* and *Forssell et al. (1997)*. It makes use of at least three different carrier-frequencies and is able to fix the ambiguities directly, without using the information of the positioning filter. This means that no ambiguity search is necessary with this type of approach.

- **MCAR:** This is a generalised approach of TCAR using more than three carrier frequencies. Considering the availability of more frequencies, the approach will be more robust (more integer) than TCAR alone. Again, no ambiguity search is needed here.

Generally, when talking of these high-accurate position solutions, differencing techniques have to be applied. So, in all the following analyses it is assumed that there are a differential reference station and its measurements available. Only brief deliberations are given for each case, when there is no reference station available at all.

Of course, usage of a differential reference station requires the transmission of the reference station data to the user, which needs some time. In the following this latency is not considered. The obtainable accuracy will appropriately be reduced dependent on the relation between this latency, the user dynamics, data rate and positioning filter design. However, this is a general effect that is independent on which of the above approaches is taken.

All carrier-phase approaches have with respect to the standard (code-only) DGPS the following quality features in common:

#	Advantage / Disadvantage	Eval.	Mitigation
1	Dramatic accuracy improvement (Carrier-phase	+++	None necessary

	accuracies obtained)		
2	Continuity reduced (After a signal outage, the ambiguities have to be resolved again)	-	None
3	Availability reduction (Carrier-phase measurements must be available for all used frequencies)	-	Possible, when more than three frequencies are available
4	No integrity (But long-term plausibility checks possible)	---	None known to date

Table 1: Carrier-phase quality features

In the following the individual approaches are generically analysed in detail.

Float RTK

The starting points for the float RTK approach are the standard double differenced observation equations for code ranges and carrier phases. They can be expressed as:

$$\nabla\Delta R_{rs}^{ij} = \nabla\Delta\rho_{rs}^{ij} + \nabla\Delta\varepsilon_{rs}^{ij} \quad (1)$$

$$\lambda\nabla\Delta\Phi_{rs}^{ij} = \nabla\Delta\rho_{rs}^{ij} + \lambda\nabla\Delta N_{rs}^{ij} + \nabla\Delta\nu_{rs}^{ij} \quad (2)$$

Hereby, the following variables are used:

$\nabla\Delta R_{rs}^{ij}$ measured code range,

$\nabla\Delta\Phi_{rs}^{ij}$ measured carrier-phase,

$\nabla\Delta\rho_{rs}^{ij}$ geometrical range,

$\nabla\Delta\Phi_{rs}^{ij}$ integer ambiguity,

$\nabla\Delta\varepsilon_{rs}^{ij}$, $\nabla\Delta\nu_{rs}^{ij}$ noise on measurements.

All variables have been double differenced as indicated by the upper satellite indices i and j and the lower receiver indices r and s.

Obviously, as in each equation four measurements are contributing, the accuracies of these measurements are twice as high as for the individual involved measurements.

$$\begin{aligned} Var(\nabla\Delta R_{rs}^{ij}) &= Var(\nabla\Delta\varepsilon_{rs}^{ij}) \\ &= 4\cdot Var(\varepsilon) \end{aligned} \quad (3)$$

$$\begin{aligned} \sigma_{\nabla\Delta R_{rs}^{ij}} &= \sqrt{Var(\nabla\Delta R_{rs}^{ij})} \\ &= 2\cdot\sqrt{Var(\varepsilon)} \\ &= 2\cdot\sigma_{\varepsilon} \end{aligned} \quad (4)$$

Analogously:

$$\begin{aligned} \sigma_{\nabla\Delta\Phi_{rs}^{ij}} &= \sqrt{Var(\nabla\Delta\Phi_{rs}^{ij})} \\ &= 2\cdot\sqrt{Var(\nu)} = 2\cdot\sigma_{\nu} \end{aligned} \quad (5)$$

In practice the double differencing is performed by assigning one of the satellites (presumably the one with highest elevation) as the reference satellite. The differencing is then performed between both stations and between any one satellite and the reference satellite.

After linearisation of the observation equation, the following (or a similar) filter design may be employed:

The state vector can be written as:

$$x = \begin{pmatrix} x_r \\ y_r \\ z_r \\ \nabla\Delta N_{rs}^{1} \\ \vdots \\ \nabla\Delta N_{rs}^{n-1} \end{pmatrix} \quad (6)$$

Hereby, x_r, y_r and z_r are the receiver WGS-84 position coordinates.

The design matrix is then:

$$H = \begin{pmatrix} H_{x,1} & H_{y,1} & H_{z,1} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ H_{x,n-1} & H_{y,n-1} & H_{z,n-1} & 0 & \cdots & 0 \\ H_{x,1} & H_{y,1} & H_{z,1} & \lambda & & 0 \\ \vdots & \vdots & \vdots & & \ddots & \\ H_{x,n-1} & H_{y,n-1} & H_{z,n-1} & 0 & & \lambda \end{pmatrix} \quad (7)$$

where the components are defined as follows:

$$H_{x,i} = -\left(\frac{x^i - x_r}{\rho_r^i} - \frac{x^n - x_r}{\rho_r^n}\right)$$

$$H_{y,i} = -\left(\frac{y^i - y_r}{\rho_r^i} - \frac{y^n - y_r}{\rho_r^n}\right) \quad (8)$$

$$H_{z,i} = -\left(\frac{z^i - z_r}{\rho_r^i} - \frac{z^n - z_r}{\rho_r^n}\right)$$

The first n-1 rows of the design matrix correspond to the code range measurements, while the remaining n-1 rows correspond to the carrier-phase measurements.

The accuracy obtained from the float RTK position solution mainly depends on the number of available measurements and frequencies (and noise) as well as on the satellite geometry.

Fixed RTK

Based on the double difference observations as already presented in the last section and after linearisation of the equation system, one obtains the canonical form

$$\mathrm{z} + \mathrm{v} = \mathrm{Hx} \quad (9)$$

Hereby, H is the corresponding observation matrix, x is the vector of unknowns, z is the vector of observations and v is the vector of measurement errors.

Solving this system for its unknowns (position and ambiguity states) leads to:

$$\hat{\mathrm{x}} = \mathrm{N}^{-1}\mathrm{c} = (\mathrm{H}^{\mathrm{T}}\mathrm{R}^{-1}\mathrm{H})^{-1}\mathrm{H}^{\mathrm{T}}\mathrm{R}^{-1}\mathrm{z} \quad (10)$$

Herein

$$\mathrm{N}^{-1} = (\mathrm{H}^{\mathrm{T}}\mathrm{R}^{-1}\mathrm{H})^{-1} \quad (11)$$

and

$$\mathrm{c} = \mathrm{H}^{\mathrm{T}}\mathrm{R}^{-1}\mathrm{z} \quad (12)$$

The quadratic sum of the residuals is further:

$$\begin{aligned}\Omega &= (z - H\hat{x})^T R^{-1} (z - H\hat{x}) \\ &= z^T R^{-1} z + z^T R^{-1} H\hat{x}\end{aligned} \quad (13)$$

Now the integer constraints are introduced into the equation system:

$$\mathrm{Kx} = \kappa \quad (14)$$

The solution to this new (increased equation system) now results in:

$$\hat{\mathrm{x}}_1 = \hat{\mathrm{x}} + \mathrm{N}^{-1}\mathrm{K}^{\mathrm{T}}\mathrm{S}_2(\kappa - \mathrm{K}\hat{\mathrm{x}}) \quad (15)$$

where

$$\mathrm{S}_2 = (\mathrm{K}\mathrm{N}^{-1}\mathrm{K}^{\mathrm{T}})^{-1} \quad (16)$$

The sum of the residuals (including the additional constraints) is now:

$$\Omega_1 = \Omega + (\kappa - \mathrm{K}\hat{\mathrm{x}})^{\mathrm{T}}\mathrm{S}_2(\kappa - \mathrm{K}\hat{\mathrm{x}}) \quad (17)$$

This shows that the overall residual is the sum of a base residual combined with the appropriate residual for the integer constraints.

$$(\kappa - K\hat{x})^T S_2 (\kappa - K\hat{x}) \quad (18)$$

This is the starting point of the integer ambiguity resolution process.

The quadratic form represents a hyperspace ellipsoid centred on the “float” solution (without the integer constraints). The shape of this ellipsoid is given by the covariance matrix. For short observation periods and in case of double differenced observations (the standard case) this ellipsoid is strongly lengthened due to the correlations between the measurements. For this reason, a decorrelation of the ambiguities can be performed (e.g. Lambda method in *Teunissen et al. (1995)*). Finally the ambiguity search is performed (e.g. *Euler and Landau (1992)*).

Independently on how the integer ambiguity search is implemented in detail, the result is the minimization of (18) taking into account the integer restrictions.

Performance Analysis for GPS

If the ambiguity fixing could be performed successfully (with at least four double difference measurements), the position accuracy is in the centimetre-range (i.e. the standard deviation of position error components will be around 1 cm).

The number of measurements or frequencies involved is not contributing significantly to a further improvement of the accuracy. Instead additional measurements or frequencies can increase the robustness of the solution and success rate significantly.

Performance Analysis for GALILEO

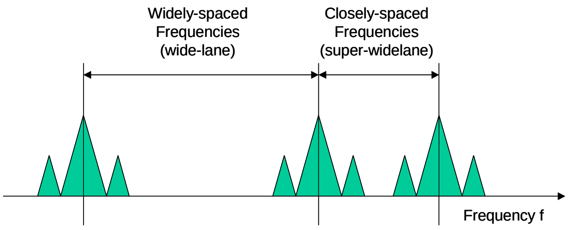
In case of GALILEO, the same statements can be given as for the GPS case. The individual and different signal structures have only negligible impact on further accuracy improvements. However, in the first place the success rate (robustness) will increase dramatically, when several frequencies are used in the approach.

TCAR

Review of the TCAR Approach

The TCAR (three-carrier ambiguity resolution) method makes use of the so-called “gap-bridging” concept, to resolve the carrier phase integer ambiguities. Two closely spaced and two widely spaced carrier frequencies are used.

The following figure shows the situation:



In contrast to standard RTK techniques, where the covariance and interdependencies of the individual line-of-sight ambiguities is taken into account, TCAR is a pure line-of-sight method for fixing single carrier-phase ambiguities.

In principle, the method can be applied to the (singly or doubly) differenced measurements (between the user and a reference station) as well as to the absolute measurements. Due to atmospheric effects we propose to use only double differenced measurements as a basis. Later on simulations will be performed to investigate the performance differences. In the following mathematical description the double difference operator is omitted for clarity.

The TCAR approach is as follows:

<u>Step 1:</u>

One code range measurement is used to obtain a first estimation of the satellite to receiver line-of-sight range. This first estimation typically has the high code noise (including other disturbing effects, like ionosphere, troposphere, multipath etc.). However, it is (hopefully) accurate enough, that it allows for the fixing of the so-called super-widelane ambiguity that is obtained from carrier-phase measurements of two closely spaced frequencies. The simplified observation equations (e.g. after double differencing and omitting double difference indication) leaves:

$$\begin{aligned} R_i &= \rho + \varepsilon_{R_1} \\ \lambda_i \Phi_i &= \rho + \lambda_i N_i + \nu_{\Phi_1} \end{aligned} \tag{19}$$

Step 1, hence, results in:

$$\hat{\rho}_{(I)} = R_1 \tag{20}$$

The accuracy of this estimator is simply the code accuracy of this code measurement, that is dependent on the bandwidth B_L, the correlator spacing d (presumed to be identical to one), the receiver integration time T, the code chip-length T_c and the signal-to-noise ratio C/N_0. According to *Van Dierendonck (1992)*, it can be expressed as follows:

$$\sigma_{R_1}^2 = \frac{B_L d}{2C/N_0}\left[1 + \frac{2}{(2-d)C/N_0 T}\right] T_c^2 \tag{21}$$

<u>Step 2:</u>

In a second step, the obtained estimation for ρ can be used together with the measured carrier phase difference $\Phi_1 - \Phi_2$ to fix the super-widelane ambiguity $N_{12} = N_1 - N_2$:

$$\hat{N}_{12} := \Phi_1 - \Phi_2 - \left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)\hat{\rho}_{(I)} \tag{22}$$

As the super-widelane signal has a wavelength of $\frac{\lambda_1 \lambda_2}{\lambda_2 - \lambda_1}$, which is (hopefully) much bigger than the code noise, N_{12} can easily be fixed by rounding to the nearest integer: $\overline{N}_{12} := [\hat{N}_{12}]$. The important aspect is now, that equation (4.4.1-6) can be turned around making use of this fixed ambiguity to obtain an improved second estimation for ρ:

$$\hat{\rho}_{(II)} = \frac{\lambda_1 \lambda_2}{\lambda_2 - \lambda_1}\left(\Phi_1 - \Phi_2 - \overline{N}_{12}\right) \tag{23}$$

The magnitude of its accuracy now is in the range of centimetres, because only carrier phase measurements have been used in this computation. Obviously, however, it was assumed that $\overline{N}_{12}$ was fixed to the correct value and its variance was eliminated this way.

$$\sigma^2_{\hat{\rho}_{(II)}} = \left(\frac{\lambda_1 \lambda_2}{\lambda_2 - \lambda_1} \right)^2 \left(\sigma^2_{\Phi_1} + \sigma^2_{\Phi_2} \right) \tag{24}$$

This accuracy now is sufficient, to fix the widelane ambiguities in the next step.

Step 3:

With this more accurate range the same procedure can be repeated for the usual widelane signal (for the two widely spaced frequencies):

$$\hat{N}_{13} := \Phi_1 - \Phi_3 - \left(\frac{1}{\lambda_1} - \frac{1}{\lambda_3} \right) \hat{\rho}_{(II)} \tag{25}$$

Hereby, equations (25) and (22) are statistically independent. Again an estimator for N_{13} can easily be found by simple rounding to the next integer, because the considered widelane signal has a virtual wavelength of $\frac{\lambda_1 \lambda_3}{\lambda_3 - \lambda_1}$, which is well above the noise of (23). The new range estimation results in:

$$\hat{\rho}_{(III)} = \frac{\lambda_1 \lambda_3}{\lambda_3 - \lambda_1} \left(\Phi_1 - \Phi_3 - \overline{N}_{13} \right) \tag{26}$$

For the accuracy of this estimation – using the same assumptions as before, one obtains:

$$\sigma^2_{\hat{\rho}_{(III)}} = \left(\frac{\lambda_1 \lambda_3}{\lambda_3 - \lambda_1} \right)^2 \left(\sigma^2_{\Phi_1} + \sigma^2_{\Phi_3} \right) \tag{27}$$

This variance now lies well below the wavelength of one of the basic frequencies, so that in a final step this integer carrier phase ambiguity may now be resolved directly.

Step 4:

The estimation for N_1 is now:

$$\hat{N}_1 = \Phi_1 - \frac{\hat{\rho}_{(III)}}{\lambda_1} \tag{28}$$

Again; turning around the equation after the fixing of N_1 to $\overline{N}_1 := [\hat{N}_1]$ yields another improved range estimation:

$$\hat{\rho}_{(IV)} = \lambda_1 \left(\overline{N}_1 - \Phi_1 \right) \tag{29}$$

When there are three or more double difference ambiguities fixed, the user position is determined extremely precise (range of millimetres to a few centimetres).

An important pre-condition for the performance of this algorithm is that there are no or only small multipath effects present, because otherwise, these systematic effects will destroy this „gap-bridging“ concept.

The question that is addressed in the following sections is, whether and under what circumstances TCAR will work for GPS and Galileo and how it will perform.

Dependent on the appropriate noise characteristics and the involved virtual wavelength, the chance for correct ambiguity fixing in each step can be given. Assuming a certain white Gaussian noise in the estimator and denoting the measurement error with x, the involved wavelength with λ and the accuracy of the estimator with σ_x this probability can be given as follows:

$$\begin{aligned} P(\text{"correct fixing"}) &= P\left(x < \tfrac{\lambda}{2}\right) \\ &= P\left(z < \frac{\lambda}{2 \cdot \sigma_x} \right) \end{aligned} \tag{30}$$

Hereby, z is defined as the normalised counterpart of x. Provided the values for the wavelength and measurement accuracy are known, the values can be obtained from the table of the standard Gaussian distribution.

As can be seen from the tables within the next sub-sections, however, the theoretical values for carrier-phase noise are too optimistic, when used directly (see e.g. *Werner et al., 1998*). The reason here is that other effects (especially multipath) are of higher magnitude.

A generic carrier-phase multipath noise effect of 0.2 cm (one sigma) has therefore been used in all following analyses. For the code measurements, a multipath error of 10 cm (one sigma) has been assumed for GALILEO E5ab (due to its high bandwidth), while in all other cases 30 cm have been assumed. When the analyses are applied to certain application environment this value can be adapted up-or downwards appropriately. For a detailed analysis of multipath characteristics, please refer to (*Van Nee, 1993, 1995).*

Performance Analysis for GPS

Table 2 and 3 show the modernized GPS signal characteristics and the virtual wavelengths and noise statistics of the frequency combinations:

Note that all carrier phase noises in these tables are given in the non-differenced mode. For double differencing, these values have to be multiplied by a factor of two.

Signal	Carrier [MHz]	Wavelength [m]	Bandwidth [MHz]	Code Noise [m]	Carrier Noise [mm]
L1	1575.42	0.1903	16	0.430	0.760
L2	1227.6	0.2442	16	0.430	0.970
L5	1176.45	0.2548	24	0.114	1.020

Table 2: Modernised GPS signal characteristics

Combined Signal	Virtual Wavelength [m]	Virtual Frequency [MHz]	Carrier Noise (without multipath) [mm]	Carrier Noise (with multipath) [mm]
L1 – L2	0.8619	347.8200	4.855	16.314
L1 – L5	0.7514	398.9700	4.249	14.074
L2 – L5	5.8610	51.1500	33.050	99.527

Table 3: Virtual wavelengths, frequencies and noise for combined modernised GPS signals

Option	Code	Super-widelane	Widelane	Base Signal
#1	L1	L2-L5	L1-L2	L5
#2	L1	L2-L5	L1-L5	L5

Table 4: Options for TCAR with modernised GPS signals

Option	Success Rate for Fixing of Super-widelane [%]	Success Rate for Fixing of Widelane [%]	Success Rate for Fixing of Base [%]	Success Rate Overall [%]
#1	100.0	97.0	100.0	97.0
#2	100.0	94.1	100.0	94.1

Table 5: Success rates for TCAR with modernised GPS signals

Signal	Carrier [MHz]	Wavelength [m]	Bandwidth [MHz]	Code Noise (1 sigma) [m]	Carrier Noise (1 sigma) [mm]
E5a	1176.45	0.2548	24.552	0.114	1.02
E5b	1207.14	0.2483	24.552	0.114	0.99
E5ab	1191.795	0.2515	51.150	0.030	0.71
E6	1278.75	0.2344	12.276	0.229	0.94
L1	1575.42	0.1903	16.368	0.176	0.76

Table 6: Galileo signal characteristics

Combined Signal	Virtual Wavelength [m]	Virtual Frequency [MHz]	Carrier Noise (without multipath) [mm]	Carrier Noise (with multipath) [mm]
L1 – E6	1.0105	296.67	5.719	19.375
L1 – E5b	0.8140	368.28	4.593	15.344
L1 – E5a	0.7514	398.97	4.249	14.074
L1 – E5ab	0.7815	383.625	3.821	14.119
E6 – E5b	4.1865	71.61	23.670	72.778
E6 – E5a	2.9305	102.3	16.603	50.564
E6 – E5ab	3.4477	86.955	16.905	56.999

E5b – E5a	9.7684	30.69	55.183	165.025
E5b – E5ab	19.5368	15.345	95.426	315.635
E5ab – E5a	19.5368	15.345	95.687	312.903

Table 7: Virtual wavelengths, frequencies and noise for combined Galileo signals

Option	Code	Super-widelane	Widelane	Base Signal
#1	E5ab	E5b–E5a	E6-E5a	E5ab
#2	E5ab	E5b-E5a	E6-E5b	E5ab
#3	E5ab	E5b-E5a	L1-E5a	E5ab
#4	E5ab	E5b-E5a	L1-E5b	E5ab
#5	E5ab	E5ab-E5a	E6-E5a	E5ab
#6	E5ab	E5ab-E5a	E6-E5b	E5ab
#7	E5ab	E5ab-E5a	L1-E5a	E5ab
#8	E5ab	E5ab-E5a	L1-E5b	E5ab
#9	E5ab	E5b-E5ab	E6-E5a	E5ab
#10	E5ab	E5b-E5ab	E6-E5b	E5ab
#11	E5ab	E5b-E5ab	L1-E5a	E5ab
#12	E5ab	E5b-E5ab	L1-E5b	E5ab
#13	E5ab	E6-E5ab	L1-E5ab	E5ab
#14	E5ab	E6-E5ab	L1-E6	E5ab
#15	E5a	E6-E5a	L1-E5a	E5a
#16	E5a	E6-E5a	L1-E6	E5a
#17	E5b	E6-E5b	L1-E5b	E5b
#18	E5b	E6-E5b	L1-E6	E5b

Table 8: Options for TCAR with Galileo signals

Option	Success Rate for Fixing of Super-widelane [%]	Success Rate for Fixing of Widelane [%]	Success Rate for Fixing of Base [%]	Success Rate Overall [%]
#1	100.0	100.0	78.6	78.6
#2	100.0	100.0	61.2	61.2
#3	100.0	74.5	100.0	74.5
#4	100.0	78.2	78.6	61.5
#5	100.0	98.1	78.6	77.1
#6	100.0	100.0	61.2	61.2
#7	100.0	45.2	100.0	45.2
#8	100.0	48.5	100.0	48.5
#9	100.0	98.0	78.6	77.0
#10	100.0	100.0	61.2	61.2
#11	100.0	44.8	100.0	44.8
#12	100.0	48.1	100.0	48.1
#13	100.0	100.0	100.0	100.0
#14	100.0	100.0	99.9	99.9
#15	92.3	100.0	100.0	92.3
#16	92.3	100.0	100.0	92.3
#17	98.9	99.5	100.0	98.3
#18	98.9	100.0	99.9	98.7

Table 9: Success rates for individual signal combinations for Galileo TCAR

Useful options for TCAR application are given in table 4. Table 5 then lists the combinations that are possible for an application of TCAR to GPS using the above signal structure. The success rates are given in table 6.

From this table, the following main conclusions may be drawn:

- The super-widelane as well as the baseline ambiguities can be fixed quite safely, while the bridging from super-widelane to widelane seems to be the most critical step.
- The overall performance is in the range of about 97.0 % (under the assumptions that have been taken here).

Of course, it must be kept in mind that the results here are strongly dependent on the assumptions made on the signal structures and carrier-to-noise ratios as well as the multipath environment. However, the assumptions for GPS here are comparable to the assumptions taken in the next section about GALILEO, so the results can be directly compared and relative results will keep their validity.

Performance Analysis for GALILEO

Tables 6 and 7 show the baseline GALILEO signal characteristics (at 42 dB-Hz) and the obtained combined signals. Note that all carrier phase noises in the above tables are given in the non-differenced mode. For double differencing, these values have to be multiplied by two. The bandwidth given here is not the fully transmitted signal bandwidth, but the tracking bandwidth assumed of an appropriate receiver. The last two combinations of table 7 are listed for completeness only. Due to inter-dependencies these combinations must be considered with caution. (The correlations between the signals E5ab, E5a and E5b have not been considered in detail here.)

Table 8 lists the optimal combinations possible for GALILEO TCAR using the above signal structure. Most of the relevant combinations start with the wideband signal E5ab, because this code range measurement will provide the best first step solution for TCAR. For the second step (super-widelane) signals are chosen that provide a long virtual wavelength (at least about 10 meters). Then for the widelane in principle all combinations with one to a few meters wavelength are considered as far as only three frequencies are used in total (where E5ab is counted as either E5a or E5b).

The following table lists the success rates for TCAR based on the signal structure above:

From this table a certain number of conclusions may be drawn:

- Obviously, the code accuracy of E5ab does allow for a (relatively) safe correct fixing of any super-widelane ambiguity.
- The last four options, which do not make use of the E5ab code range for the first step, however, have a high success rate for the remaining two fixing steps (widelane and base frequency ambiguity fixes).
- Option #13 seems to be best under the used assumptions.
- Compared to GPS TCAR, in GALILEO there are combinations with higher success rates.

MCAR

Review of the MCAR Approach

The term MCAR (“multiple [frequency] carrier [phase] ambiguity resolution”) is quite new, meaning a generalisation of the TCAR approach to more than three frequencies. Up to now there is no fixed algorithm working optimally on the increased number of frequencies. Several possibilities are available for exploiting the increased measurement redundancy.

Considering the TCAR-like approach, these possibilities are:

- **Code-noise reduction:** The code noise on the individual measurements can be reduced by e.g. taking the mean of the different code measurements. This will remove the code noise to some extent, but will not remove other ill effects, like atmospheric effects. Of course, multipath (which is one of the most important effects) will only be reduced to some small extent, because it is typically very similar on all code measurements when a similar bandwidth is available. When there is one code with high bandwidth, this code should anyway be better than all other code measurements and should be used in the first TCAR/MCAR step.
- **Fine gap-bridging:** Usage of a finer bridging between the different virtual signals could lead to performance improvements. For example, after a super-widelane signal (wavelength e.g. around 10 m) has been fixed, an intermediate widelane of about 4 m could be fixed more safely than a 1 m widelane signal. After the intermediate 4 m fix, the 1 m widelane might be fixed. Of course there will be several dependencies involved and intensive simulations seem to be necessary to characterise these effects.
- **Consistency checks:** Usage of different (statistically as independent as possible) combinations for the super-widelane or widelane steps to check consistency of the ambiguity fixings.

It is clear, that the only advantage of this approach will be in the robustness (integrity) of the solution. Considering the accuracy, there will not be a significant improvement, because TCAR alone – provided its fixing steps are working correctly – already leads to centimetre level position accuracy. Availability and continuity will only scarcely be impacted, because if there is a line-of-sight problem, it will occur to all signals coming from the satellite not only to one or two. However, if there are issues with the signal generation in the satellite or signal reception at the receiver for a certain frequency, a small gain is obviously involved.

In the following a brief assessment of the obtainable performances is done.

Performance Analysis for GPS

For GPS, there will only be three frequencies available. No MCAR can be applied with only the available three frequencies. Hence no further assessment is given here.

Performance Analysis for GALILEO

Considering item 1 of above (code smoothing among frequencies), its noise removing effects will be nearly negligible with respect to the other systematic effects, when the GALILEO codes are employed. Consistency checks can be done on position level and are not further analysed here.

As there are more than three carriers available, several different wide-lanes (or super-wide-lanes) can be constructed.

Using the following signal combinations in a sequence for individual TCAR steps a "fine-gap-bridging" can be realised:

(E5ab) / E5b-E5a / E6-E5b /E6-E5a / L1-E6 /L1-E5a / L1

These signal combinations result in the following virtual wide-lane wavelengths:

9.7684 m / 4.1865 m / 2.9305 m / 1.0105 m / 0.7514 m / 0.19 m

Analysis has shown that with this application the probability of success in this case is close to 100 % ($> 1 - 10^{-5}$).

CONCLUSIONS

The carrier phase techniques have been reviewed briefly. The TCAR (and MCAR) methods have been analysed theoretically. Optimal signal combinations for application of TCAR have been derived.

However, what still needs to be done is a thorough comparison between the float and fixed RTK methods versus the TCAR/MCAR methods.

Even though the TCAR and MCAR methods show high success rates, when they are based on a good code range signal such as the wide-band E5ab, they do not exploit any geometrical information like float or fixed ambiguity techniques. Hence, the performance that can be expected from TCAR or MCAR cannot be better than the performance that can be derived from RTK techniques. Nevertheless, these techniques have a high success rate on each individual line-of sight. The success rates given in the present paper each is related to an individual line-of sight. Note also, that multipath effects can deteriorate the success rates significantly. Werner et. al. (1998a) shows also that the performance is also strongly dependent on the code accuracy of the first TCAR step, and demonstrates what is the result after any of the intermediate fixing steps fails.

Furthermore, for GALILEO TCAR the following conclusions could be drawn and are repeated here:

- Obviously, the code accuracy of E5ab does allow for a (relatively) safe correct fixing of any super-widelane ambiguity.
- The last four options, which do not make use of the E5ab code range for the first step, however, have a high success rate for the remaining two fixing steps (widelane and base frequency ambiguity fixes).
- Option #13 with the signal combinations E5ab, E6-E5ab, L1-E5ab, E5ab seems to be best under the used assumptions.
- Compared to GPS TCAR, in GALILEO there are combinations with higher success rates.

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