

# An Integrity Monitoring Technique for Multiple Failures Detection

Ilaria Martini

*IfEN Gesellschaft für Satellitennavigation mbH (IfEN GmbH), D-85586 Poing, Germany*

Guenter W. Hein

*Institute of Geodesy and Navigation, University FAF Munich, D-85577 Neubiberg, Germany*

**Abstract.** Within the next future, the advent of Galileo, GPS modernization and of the GNSS augmentation systems, will lead to a rapid development of GNSS receiver technologies. It is expected that the improved accuracy performance will extend the use of satellite positioning and navigation to applications where present systems do not fulfill user integrity requirements and do not allow the receiver certification.

In this scenario a central role is played by the integrity monitoring capability. In fact a large part of users is carrying out applications in which an error in might represent an excessive risk, in particular when human lives are involved. For these applications, the system capability of protecting the user against system failure is of primary importance. The main example is given by aeronautical applications where at present the fulfillment of integrity requirements during approaches of type CAT I (and higher) has still to be reached.

In this context, it is essential for the user to take advantage of Receiver Autonomous Integrity Monitoring (RAIM) techniques. In fact, although regional integrity is provided by space-based augmentation systems (SBAS) like EGNOS, WAAS and MSAS and global integrity will be transmitted by the European Galileo satellite navigation system in the near future, RAIM is the only technique able to monitor receiver local errors. Since it is located at the end of the integrity processing chain, its role is essential in the integrity determination process.

It is also highlighted that present RAIM techniques have limitations, which in particular jeopardize the possibility to certificate satellite navigation receiver as sole or stand alone positioning platform in aeronautical applications. The main limitation is represented by the fact that all present RAIM techniques protect users only against one single failure affecting a particular satellite range measurement. Multiple failure events are usually assumed to have a very low probability of occurrence. But since in safety of life applications the continuity and availability requirements in terms of probability of missed detection are very strict, the multiple failure events need more attention and cannot be disregarded anymore by RAIM techniques.

This paper presents an investigation on present RAIM technique and their performance with respect to multiple failures. Furthermore it presents a technique able to overcome the present integrity monitoring limitations and to protect the user receiver in case of multiple failures.

## I. INTRODUCTION

The first purpose of the study presented in this paper is to analyze the performance of present integrity monitoring algorithms in presence of multiple simultaneous range errors. In particular the analysis has focused on Least Squares Integrity Algorithm, commonly considered as the reference of

RAIM algorithms [1]. An accurate study of the Least Squares Integrity Algorithm characteristics allows to derive important properties of its performance.

It is demonstrated that the Least Squares Integrity Algorithm performs a projection into a subspace and then its observation is limited because it loses a certain amount of information with every epoch. The fact that it discards a part of information identifies the problem of receiver integrity monitoring with respect to multiple failures detection capability.

Theoretical analysis and simulation results are reported in this paper to show the Integrity Algorithm limitations in cases of multiple failures due to mutual cancellation of the errors. The transformation performed by Least Squares Integrity Algorithm has been analyzed in order to explicitly express the component lost by the algorithm. This has been the starting point for a reformulation of the integrity problem.

In fact the second part of the study described in this paper presents an integrity monitoring technique able to reconstruct the component discarded by Least Squares Integrity Algorithm. The presented technique aims to protect the user receiver against any kind of errors without limiting to single failure cases. In particular it is able to detect multiple satellite failures, which are not detected by the present integrity monitoring techniques.

The proposed technique uses measurements collected on different epochs. It provides a test quantity to be compared with a threshold analogous to the Least Squares Integrity Algorithm but is able to detect all kind of failures. Simulation results are presented to show the improvement of the detection capability.

This technique aims to provide also an estimation of the error for each satellite. In fact the Least Squares Integrity Algorithm is based on the test statistic computed using the residuals. Each residual does not contain information on the bias affecting the corresponding line of sight. In general errors are spread among the different residuals. Each residual contains also information on the errors affecting the other lines of sight.

The proposed technique aims to recover the information for each line of sight. This important capability on one side constitutes the identification capability of this technique (once a failure is detected, the algorithm identifies the line of sight affected by the bias), on the other hand it provides also an estimation of the error affecting the single line of sight.

Simulation results show the performance of the algorithm with respect to identification and error estimation.

As shown and discussed in the paper, the algorithm performance is still under investigation because it is very sensitive with respect to the magnitude of error. In particular different techniques to solve the ill-conditioning of the problem are under investigation in order to make the algorithm more robust with respect to error changes and to reduce the minimum detectable error, which at the present may have a magnitude of hundreds of meters.

It is important to mention that the algorithm introduces a delay. This delay is deterministic and depends on the number of satellites in view in each epoch. It is demonstrated and described in the paper that the delay is around 2 epochs and in any case less than 5 epochs (corresponding to 5 seconds with a processing performed every second). Considering the aeronautical user requirements in terms of time to alarm (TTA), this aspect is not critical since, for example, the CAT I user requirement assumes a TTA up to 6s.

Simulations are carried out where the proposed approach is verified with data from a GPS constellation introducing for each scenario different combinations of feared events. In particular, simulations are performed first with the hypothesis that the errors of each line of sight are constant during the number of consecutive epochs collected by the algorithm to provide each output. This hypothesis is taken only in an intermediate step, because it allows explaining the RAIM characteristics with respect to the proposed approach. Then it is removed to analyze the algorithm performance in more realistic cases.

In conclusion the paper shows that this technique enables the user to detect multiple satellite failures, even in those cases where present RAIM technique cannot detect the occurrence of an anomalous situation. Furthermore this new approach is able to identify which are the failing satellites and to estimate for each of them the actual bias affecting the pseudorange measurement.

Further investigation will be carried out in order to test the algorithm performance in dynamic cases and to improve the performance with respect to minimum detectable error magnitude.

## II. NAVIGATION ALGORITHM

This section introduces the problem of integrity monitoring recalling briefly the user navigation algorithm, which is in the receiver processing chain the step preliminary to the integrity algorithm. The description has the main scope to present the notations used in the paper.

The pseudorange measurements  $y^i(k)$  collected and processed by the user receiver at epoch  $k$  can be modeled with the following expression

$$y^i(k) = \rho^i(k) + c\Delta t_u(k) + e_i(k) \quad (1)$$

where  $\rho^i(k)$  is the geometric range between the user antenna and satellite  $i$ -th antenna at epoch  $k$ -th,  $\Delta t_u(k)$  is the user

receiver clock offset with respect to the Reference System Time and  $c\Delta t_u(k) = b_u(k)$  with the speed of light  $c$  is the receiver clock offset expressed in meter,  $e_i(k)$  is the sum of measurement errors at satellite level (satellite positions and clock errors), at propagation path level (ionospheric and tropospheric delay), and at receiver level (multipath, interference, receiver noise).

Collecting observations from  $N$  satellites and linearizing around an initial estimate  $\widehat{X}_0(k)$ , of the unknown vector with user position and clock  $\overline{X}(k) = [x_{user}, y_{user}, z_{user}, b_u]$ , the following system is obtained

$$\overline{\Delta Y}(k) = \overline{H}(k) \cdot \overline{\Delta X}(k) + \overline{E}(k) \quad (2)$$

where  $\overline{\Delta X}(k) = \overline{X}(k) - \widehat{X}_0(k)$  is the shift of the user unknown vector with respect to  $\widehat{X}_0(k)$ ,  $\overline{H}$  is the matrix containing the first order derivative of the line of sights with respect to each component of the unknown vector  $\overline{\Delta X}(k)$ , and  $\overline{\Delta Y}(k) = \overline{Y}(k) - \overline{Y}_0(k)$ .

The navigation algorithm provides the solution of the linear model, expressed by (2). In particular it provides the weighted least Squares solution given by

$$\overline{\Delta X}_{LS}(k) = \left[ \overline{H}' \overline{R}^{-1} \overline{H} \right]^{-1} \overline{H}' \overline{R}^{-1} \cdot \overline{\Delta Y} \quad (3)$$

where  $\overline{R}(k) = Cov(\overline{E}(k))$  is the weighting matrix represented by the covariance matrix of the error vector  $\overline{E}(k)$ .

In this study  $\overline{R}(k)$  is considered the identity matrix. This allows focusing the attention on the geometric aspects of the navigation and integrity algorithms.

Equation (6) expresses the least squares estimation of the user unknown vector, input to the integrity algorithm.

### III. LEAST SQUARES INTEGRITY ALGORITHM ANALYSIS AND CHARACTERIZATION

Scope of the integrity algorithm is monitoring the error vector  $\bar{E}(k)$ , in order to detect the occurrence of anomalous situations. Such events occur when the error vector  $\bar{E}(k)$  increases due to failures and some of the error component is affected by biases. For this scope, it would be useful to monitor each component of the vector  $\bar{E}(k)$ , since each corresponds to one different line of sight. With the ability of distinguishing among different components it would be possible to directly identify the line of sight affected by failure. Furthermore this would provide the receiver with the capability to detect and identify multiple failures. At the present, receiver integrity monitoring algorithms do not have this capability and present some limitations.

This section analyses the characteristics of Least Squares Integrity Algorithm, commonly assumed as reference for RAIM algorithm. In particular it investigates how it operates on the error vector  $\bar{E}(k)$ . This allows to clearly identifying the problems of present techniques, in particular concerning the capability of detecting multiple failures.

In the following sections the limitations of Least Squares Integrity Algorithm are described and then completed with the mathematical justification.

#### A. Least Squares Integrity Algorithm: main limitations.

Least Squares Integrity Algorithm uses the norm of the vector  $\Delta\bar{Y} = \bar{Y}(k) - (H \cdot \Delta\bar{X}_{LS}(k))$  to control the integrity of the navigation system.

A test statistic is computed each epoch using the sum of the Squares residuals, defined by

$$SSE(k) = [\Delta\bar{Y}(k)]^T \cdot \Delta\bar{Y}(k) = \|\Delta\bar{Y}(k)\|^2 \quad (4)$$

This **test statistic** is then compared with a **threshold**, computed to meet the final user requirements in terms of probability of missed detection and false alarm. When the test statistic exceeds the threshold an alarm is raised for the user.

This method is based on the fact that when the  $\|\bar{E}(k)\|$  increases, “with high probability” also  $\|\Delta\bar{Y}(k)\|$  increases.

This is always true in case of single failure, when only one of the  $N$  line of sight components presents a bias. It is also confirmed by demonstration performed with simulations. In particular it is observed that  $\|\Delta\bar{Y}(k)\|$  increases not necessarily in a proportional way with respect to  $\|\bar{E}(k)\|$ .

The impact of the bias on the test statistic is a function of the satellite-receiver mutual geometry.

The test statistic is a single scalar quantity, which informs the user on the presence of a single failure in one of the  $N$  lines of sight. The test statistic does not include the information to identify which line of sight is affected by the bias. Usually a conservative approach is used for setting the threshold and the algorithm considers the worse case corresponding to a failure in the satellite having the smallest detection capability.

It is also observed that there is no protection performed by Least Squares Integrity Algorithm in case of multiple failures. The reason, deeply investigated in the following sections, can be anticipated saying that in case of multiple biases, the errors compensate between each other's and  $\|\Delta\bar{Y}(k)\|$  might not increase, even in case of significant biases in the  $\|\bar{E}(k)\|$ 's components.

The **Least Squares Integrity Algorithm characteristics** and limitations are summarized in the following:

- a. It protects the user only in case of one **single failure**. Cases of multiple satellite failures are not detected. This represents the main limitation precluding its use as stand alone system in many applications (for example in so called “Safety of Life” aeronautical applications).
- b. It does not monitor the single components of the vector  $\bar{E}(k)$ , but the quantity  $\|\Delta\bar{Y}(k)\|$  considered as representative of the  $\|\bar{E}(k)\|$ . This corresponds to collapsing all the information of the different line of sights in one scalar quantity, **losing the capability of distinguishing each different component**. In fact an alarm corresponds to the detection of a failure “in one of the line of sight component” with a certain probability of false alarm. It does not provide any information on which is the line of sight having the failure and on its error magnitude.
- c. The approach followed by the algorithm is not based on the mathematical derivation of the relationship between  $\|\bar{E}(k)\|$  and  $\|\Delta\bar{Y}(k)\|$ . It introduces a statistical approach to estimate with which probability the  $\|\Delta\bar{Y}(k)\|$  increases with respect to  $\|\bar{E}(k)\|$ , in the worse satellite case: it considers  $\|\Delta\bar{Y}(k)\|$  as a quantity representative “in a statistic sense” of  $\|\bar{E}(k)\|$ . This restricts its validity to only the single failure case.
- d. In order to identify in which line of sight is contained a detected failure, there are different

**identification** algorithms. Without entering in detail of one of these techniques, here some characteristics are reminded.

- a. These techniques require having a **number of satellites** in view at least equal to the number of receiver unknowns plus 2. In GNSS-only receivers visibility of at least 6 satellites is required. In a Galileo + GPS combined receiver, the unknown vector contains also the Time Reference Offset and the minimum number of satellites in view required by these identification techniques is 7.
- b. These techniques identify which is the line of sight affected by failure but do not provide any information on the **biases magnitude** of the failure detected in the vector  $\bar{E}(k)$ .

The previous observations have highlighted the need to have a deep investigation. Then it has been performed an analysis on how the Least Squares Integrity Algorithm operates on the vector  $\bar{E}(k)$ .

Scope of the following section is presenting a mathematical formulation of the Least Squares Integrity processing.

In particular the vector of residuals  $\bar{\Delta Y} = \bar{Y}(k) - (\bar{H} \cdot \bar{\Delta X}_{LS}(k))$  is expressed as a function of the bias vector  $\bar{E}(k)$ , as contained in [2]. This step is important because it allows analyzing the transformation performed by Least Squares Integrity Algorithm on the vector of observation  $\bar{E}(k)$  through the monitoring of  $\|\bar{\Delta Y}(k)\|$ .

This study has been the starting point for the investigation of a new approach overcoming all the previously mentioned integrity monitoring limitations.

### B. Relationship between error vector $\bar{E}(k)$ and residuals $\bar{\Delta Y}(k)$

As presented in [2], the following relationship can be demonstrated between the error vector  $\bar{E}(k)$  and the residual vector  $\bar{\Delta Y}(k)$ :

$$\bar{\Delta Y}(k) = (I - \bar{H}\bar{S}) \cdot \bar{E}(k) \quad (5)$$

where  $\bar{S} = \left[ \bar{H}' \cdot \bar{H} \right]^{-1} \bar{H}'$  is the pseudo-inverse matrix of  $\bar{H}$ .

Equation (5) represents the transformation performed by Least Squares Integrity Algorithm on the errors vector  $\bar{E}(k)$ . In particular it is important because it shows that computing  $\bar{\Delta Y}$  is equivalent to applying the linear transformation  $(I - \bar{H}\bar{S}) : R^N \times R^N$  to the error vector  $\bar{E}(k)$ .

The Least Squares Integrity Algorithm monitors the norm of this transformed vector, as expressed by (4).

### C. Projection into a subspace

The transformation  $(I - \bar{H}\bar{S}) : R^N \times R^N$  corresponds to losing a part of information contained in the error vector  $\bar{E}(k)$ .

It can be demonstrated that the linear transformation  $I - \bar{H}\bar{S}$  from  $R^N$  to  $R^N$  projects any vector of  $R^N$  in the null space of  $\bar{H}'$ , called  $Null\bar{H}'$ . This is done considering a generic vector  $\bar{y} \in R^N$  and a vector  $\bar{x} \in R^N$ , satisfying the equation  $(I - \bar{H}\bar{S}) \cdot \bar{y} = \bar{x}$ . The vector  $\bar{x} \in R^N$  is the transformed vector of  $\bar{y} \in R^N$ , according to the transformation  $I - \bar{H}\bar{S}$ . Then it is always true that  $\bar{x} \in Null\bar{H}'$ ,  $\forall \bar{y} \in R^N$ . In fact  $\forall \bar{y} \in R^N$ ,  $\bar{H}' \cdot \bar{x} = \bar{H}' \cdot \left( I - \bar{H} \left[ \bar{H}' \bar{H} \right]^{-1} \bar{H}' \right) \cdot \bar{y} = \mathbf{0}_{N \times N}$ .

The direct consequence of this is that the Least Squares Integrity Algorithm projects  $\bar{E}(k)$  into the  $Null\bar{H}'$ .

With the scope of observing and monitoring  $\bar{E}(k)$ , the Least Squares Integrity Algorithm observes the projection of  $\bar{E}(k)$  into the  $Null\bar{H}'$ .

The  $Null\bar{H}'$  is a subspace of  $R^N$ . Like in all the projections in subspaces, some information is lost during this transformation. This means that there are some combinations of  $\bar{E}(k)$  components leading to residual vectors with norm equal to zero.

These correspond to cases of multiple failures not detected by this algorithm.

Aim of the following analysis is investigating the characteristics of this projection in order to extrapolate the detection capability of Least Squares Integrity Algorithm.

Furthermore a new approach is studied and presented where the missing information lost each epoch is reconstructed through the elaboration of consecutives epochs and taking advantage of the constellation geometry diversity.

#### D. Error vector components

The vector  $\bar{E}(k) \in R^N$  is not directly measured at receiver level and the only available observation is its projection in a subspace, obtained through the linear transformation  $(I - \bar{H}\bar{S}): R^N \times R^N$ .

Thanks to the geometry diversity of the line of sight directions, the derivatives contained in the design matrix of each epoch generate 4 columns linearly independent. This is equivalent to having  $rank \bar{H} = 4$ . In virtue of the orthogonality, the  $Null \bar{H}^t$  has dimension  $N - 4$ .

$(I - \bar{H}\bar{S})$  is then a transformation projecting any vector of  $R^N$  in the subspace  $Null \bar{H}^t$  of dimension  $N - 4$ . This projection loses of the vector  $\bar{E}(k)$  a quantity of information with 4 degrees of freedom.

It is also highlighted that the  $Null \bar{H}^t$  is a subspace of  $R^N$  orthogonal to  $Im \bar{H}$ .

Since the  $Null \bar{H}^t$  and the space  $Im \bar{H}$  are orthogonal, they represent a base of  $R^N$ . This means that each vector  $\bar{E}(k) \in R^N$ , can be expressed as the sum of the component on the  $Null \bar{H}^t$  and the component on  $Im \bar{H}$ .

Each epoch the Least Squares Integrity Algorithm discards the component of  $\bar{E}(k)$  on  $Im \bar{H}$  and it observes only the component on the  $Null \bar{H}^t$ .

The algorithm detectability depends then on the position of the vector  $\bar{E}(k)$  with respect to the subspace  $Null \bar{H}^t$  of  $R^N$ , in particular on the magnitude of the lost component with respect to the projected one.

The component lost by the algorithm is

$$C_{\_lost} = \left( \bar{E}^t(k) \cdot Im \bar{H} \right) SPAN \{ Im \bar{H} \}$$

And the Least Squares Integrity Algorithm projection reconstructs only the

$$C_{\_proj} = \left( \bar{E}^t(k) \cdot Null \bar{H}^t \right) SPAN \{ Null \bar{H}^t \}$$

where  $SPAN \{ \bar{A} \}$  represents the space generated by the column vectors of matrix  $\bar{A}$ .

The following figure shows the two different components

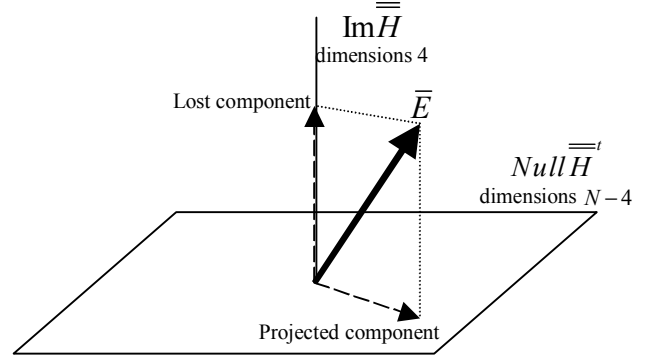


Fig. 1. Component projected and lost by least squares algorithm

#### E. Least Squares Integrity Algorithm limitations with respect to multiple failure detection

The Least Squares technique relies on the fact that the projection of  $\bar{E}(k)$  might be representative of the  $\| \bar{E}(k) \|$ . But this is not always valid. This depends on the angle between the vector  $\bar{E}(k)$  and the subspace  $Im \bar{H}$ . For those vectors, whose angle is small, the Least Squares Algorithm detectability is jeopardized.

The position of the vector  $\bar{E}(k)$  with respect to the subspace  $Im \bar{H}$  is related to the presence of multiple failures.

In [2], it is demonstrated that if  $p$  is the number of components of  $\bar{E}(k) \in R^N$  containing bias, then the space in  $R^N$  of the vectors not affecting the Least Squares Integrity test statistic has dimension  $\{ \max(0, 4 - (N - p)) \}$ . This means that in case of  $p > N - 4$ , there is a space of dimension  $p - (N - 4)$  of errors  $\bar{E}(k)$  not affecting the integrity monitoring, giving null contribution to the test statistic. Then there are situations with a number of failures bigger than  $N - 4$ , where the Least Squares Integrity Algorithm does not detect the anomaly and does not generate any alarm.

And in case of  $p \leq N - 4$ , there are cases, in which the errors give a contribution to the test statistic, but it is small with respect to threshold and also in these cases the detection fails.

## IV. A NEW INTEGRITY MONITORING TECHNIQUE

The analysis performed up to here introduces the idea to define a new approach, aiming to overcome Least Squares Integrity Algorithm limitations and to reconstruct its lost component. This section describes this approach, explaining

how it reconstructs the component on the  $\text{Im } \bar{H}$  through the elaboration of consecutive epochs.

To explain the concept it is firstly presented a two-dimensional geometrical problem. Then its application to the general integrity case is described.

#### A. Two-dimensional case.

Consider the following problem in  $R^2$ , which is equivalent to the basic integrity problem but only in a theoretical two dimensional case.

Suppose to have the problem of finding a vector  $(\bar{x}, \bar{y}) \in R^2$  having each epoch limited information. In fact each epoch it is possible to compute only one of the two vector components: for example the  $\bar{x}$  estimation is available, but not the  $\bar{y}$ . The unique measure of the  $\bar{x}$  component is not enough to re-construct the vector  $(\bar{x}, \bar{y}) \in R^2$ . Besides it is also not representative of the norm of  $(\bar{x}, \bar{y})$  (in case for example of  $\bar{y} \gg \bar{x}$ ).

Since it is available only one degree of information (only the  $x$  component) each epoch, two independent observations are necessary in order to estimate the quantity,  $(\bar{x}, \bar{y}) \in R^2$ .

From this observation comes the idea to collect the information on two consecutive epochs and adding to the input data new information on the vector  $(\bar{x}, \bar{y}) \in R^2$ . In particular each epoch the reference system changes, then the vector (assuming it constant on two consecutive epochs) is represented in the new reference as  $(\bar{x}^i, \bar{y}^i) \in R^2$ , as displayed in Fig. 2

The problem is then to reconstruct the vector  $(\bar{x}, \bar{y}) \in R^2$  knowing  $\bar{x}$  and  $\bar{x}^i$ . With these two components, it is possible to estimate accurately  $(\bar{x}, \bar{y}) \in R^2$ .

It is important to note that the two reference systems must be linearly independent: in  $R^2$  this corresponds to having  $xOy$  reference systems with axis rotated every epoch.

Fig. 2 shows the problem in the two dimensional case.

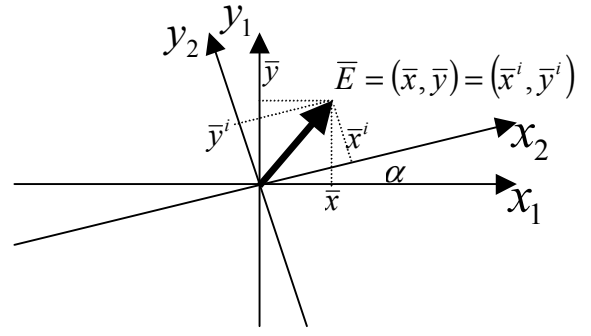


Fig. 2 Basic idea of the proposed approach in a two dimensional case.

The following system presents the mathematical formulation of the problem:

$$\begin{cases} \bar{y}^i = -\bar{x} \cdot \sin(\alpha) + \bar{y} \cdot \cos(\alpha) \\ \bar{x}^i = \bar{x} \cdot \cos(\alpha) + \bar{y} \cdot \sin(\alpha) \end{cases}$$

Form the previous equation, it is obtained  $y$  as function of  $\bar{x}$  and  $\bar{x}^i$ :

$$\bar{y} = -\frac{\cos(\alpha)}{\sin(\alpha)} \cdot \bar{x} + \frac{1}{\sin(\alpha)} \cdot \bar{x}^i$$

Concluding, it has been shown that it is possible to reconstruct a vector of  $R^2$ , from the two projections,  $\bar{x}$  and  $\bar{x}^i$ , respectively on the two reference systems  $xOy$  and  $x^iOy^i$ . Important to remember that the two reference systems must be linearly independent from epoch to epoch and that the vector has been assumed constant in the two reference system  $xOy$  and  $x^iOy^i$ .

#### B. Integrity Monitoring case

In the integrity monitoring problem, the unknown vector,  $\bar{E}(k) \in R^N$  has dimension  $N$ , and the integrity algorithm computes each epoch its projection on the  $\text{Null } \bar{H}^i$ , a subspace of dimension  $N-4$ .

Since in each projection  $N-4$  degrees of information are available, the new approach tries to reconstruct the vector  $\bar{E}(k)$ , collecting projections on the consecutive epochs, and taking advantage of the geometry diversity. In fact each epoch the satellites and the receiver move relatively and the mutual geometry changes. This corresponds to

changing the point of view for observing the vector  $\overline{E}(k)$ . This allows retrieving the missing information.

### C. Three important considerations

Three important observations must be considered at this point.

1. As already mentioned, in order to have a valid added value of information each epoch, for the estimation of the vector  $\overline{E}(k)$ , the **subspaces of different epochs must be linearly independent among each other**. The following figure shows the case with independent reference systems on two consecutive epochs:

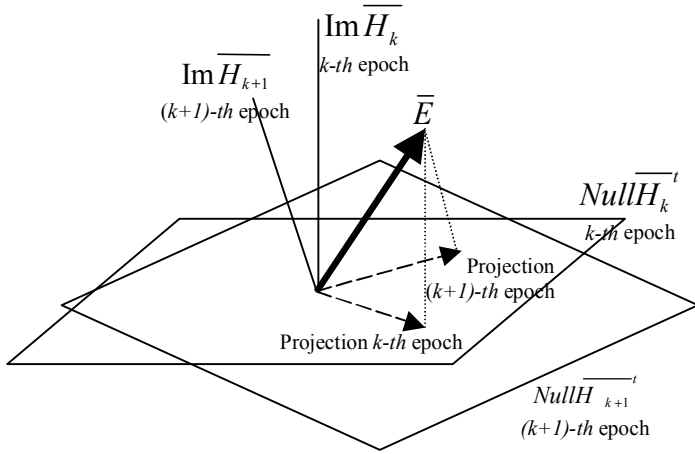


Fig. 3 Linearly independence of  $Null\overline{H}^t$  and  $Im\overline{H}$ .

Fig. 3 shows the subspaces used as basis in  $R^N$  to represent the vector  $\overline{E}(k)$  as sum of the projection and the lost component.

In particular it is displayed what should be the ideal relationship between the  $Null\overline{H}^t$  and the  $Im\overline{H}$  at epoch  $k$  and those at epoch  $(k+1)$ . A correct reconstruction requires that the  $Null\overline{H}_k^t$  at epoch  $k$  and  $Null\overline{H}_{k+1}^t$  at epoch  $(k+1)$  are independent. The same must be valid for the  $Im\overline{H}_k$  with respect to  $Im\overline{H}_{k+1}$ .

Unfortunately this is not valid in the integrity case.

The subspaces  $Null\overline{H}^t$  and  $Im\overline{H}$  do not satisfy the property of linearly independence from epoch to epoch. In fact the last column of the design matrix contains the derivative of the distance with respect to the user receiver clock offset. All the components of the last column are equal to  $-1$ . This means that subspaces  $Im\overline{H}$  of different epochs share the subspace of dimension 1, generated by the following

vector  $(-1, -1, \dots, -1)_N$ . Consequently  $Im\overline{H}$  and  $Null\overline{H}^t$  at different epochs are not linearly independent. Next section describes a new formulation of the problem in order to take into account this further degree of complexity.

2. A second aspect to be taken into account is that the geometry changes slightly, with respect to the enormous distances between receiver and satellites. This fact leads to **ill-conditioned systems to be solved**. In particular the bad conditioning of the problem is due to the following two aspects:
  - a. Small geometry rotation: this leads to matrixes difficult to be inverted, i.e. ill-conditioned matrixes.
  - b. Variation of error vector with respect to geometry variation. This is related to sensitivity of the algorithm with respect to variation of the error vector  $\overline{E}(k)$  in the epochs collected for the estimation, in particular in the number of steps necessary for the algorithm. Standard inversion methodologies are very sensitive with respect to error variations and further improvements on this aspect are still under investigation.
3. The algorithm provides the detection and error estimation with a **delay**, corresponding to the number of epochs to be collected minus one. As anticipated in the introduction the delay is **around 2 epochs** and **maximum 5 epochs**. Since each step a degree of information equal to  $N-4$  is added, the **number of epochs necessary to reconstruct the complete information**, called  $s \in Z$  must satisfy  $(N-4) \times s \geq N$ , that yields to

$$s = \left\lceil \frac{N}{(N-4)} \right\rceil^{\text{int sup}}. \text{ In a general case it is}$$

$$s = \left\lceil \frac{N}{(N - \#unknowns)} \right\rceil^{\text{int sup}}. \text{ In reality,}$$

because of the issue explained at point 1, the number of unknowns to be estimated is  $N-1$  instead of  $N$  (the reason is explained in the following section), and then the actual number of

$$\text{epoch is } n = \left\lceil \frac{N-1}{(N - \#unknowns)} \right\rceil^{\text{int sup}}. \text{ The}$$

following table gives for each number of satellites in view the number of epochs to be collected to estimate  $\overline{E}(k)$  and the  $delay = s - 1$ .

TABLE I  
NUMBER OF EPOCHS COLLECTED BY THE PROPOSED ALGORITHM FOR DIFFERENT  
NUMBER OF SATELLITES IN VIEW.

Satellites in view	Galileo (or GPS) only case (4 unknowns) Number of epochs $s = \left\lceil \frac{N-1}{N-4} \right\rceil^{\text{int sup}}$ $\text{delay} = s - 1$	Galileo +GPS case (5 unknowns) Number of epochs $s = \left\lceil \frac{N-1}{N-5} \right\rceil^{\text{int sup}}$ $\text{delay} = s - 1$
5	$s = 4, \text{delay} = 3$	-
6	$s = 3, \text{delay} = 2$	$s = 6, \text{delay} = 5$
7	$s = 3, \text{delay} = 2$	$s = 3, \text{delay} = 2$
8	$s = 2, \text{delay} = 1$	$s = 2, \text{delay} = 1$
> 8	$s = 2, \text{delay} = 1$	$s = 2, \text{delay} = 1$

#### D. New formulation of the problem: common component.

To solve the problem of linear independence of the reference systems, a new representation of the vector  $\bar{E}(k) \in R^N$  is necessary. Fig. 4 displays the followed approach:

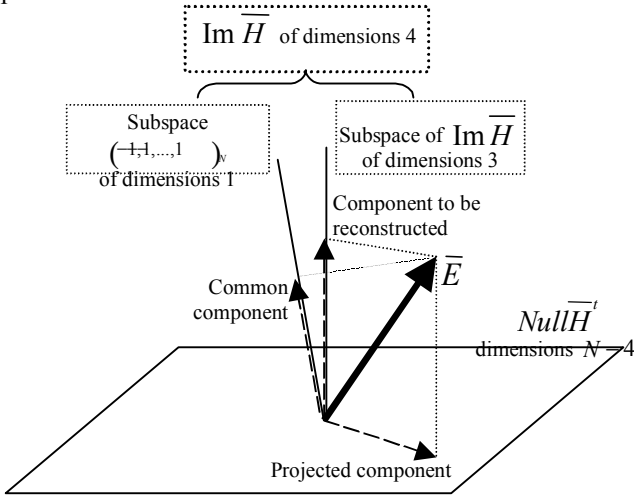


Fig. 4. Final formulation of the error vector.

The lost component of the vector  $\bar{E}(k) \in R^N$  is expressed as the combination of the component on the subspace  $(-1, -1, \dots, -1)_N$ , shared by all the design matrixes of different epochs, and the component on the space orthogonal to this subspace.

The vector  $\bar{E}(k) \in R^N$  is then represented as the combination of three components

$$\bar{E} = C\_proj + C\_lost = \\ C\_proj + C\_reconstr + C\_common$$

where  $C\_proj$  is the component on the  $Null \bar{H}^t$ ,  $C\_common$  is that on the subspace  $(-1, -1, \dots, -1)_N$ , and  $C\_reconstr$  is that on the subspace orthogonal to  $(-1, -1, \dots, -1)_N$  and to  $Null \bar{H}^t$ .

The consequence of the fact that each epoch the subspaces  $Im \bar{H}$  are not linearly independent, because they share the subspace of dimension 1, generated by the following column vector  $(-1, -1, \dots, -1)_N$  is that the component on this shared subspace cannot be estimated.

This characteristic is intrinsic to the navigation and integrity algorithms as it can be easily deduced, reflecting on the properties of the least Squares estimation. In case that all the satellites have a common bias, the navigation algorithm allocates this component to the receiver clock error.

Following figure shows two different situations, with and without the common component.

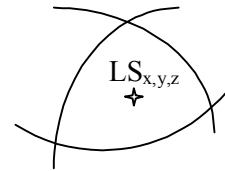


Fig. 5. Least Squares position solution with gaussian noise

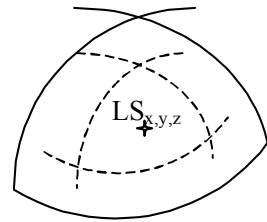


Fig. 6. Least Squares position solution with gaussian noise and a common bias in each pseudorange

The Fig. 6 shows that the least Squares solution with a common bias in each satellite pseudorange yields to the same least Squares geometrical position  $(x, y, z)$  of the case with no biases (Fig. 5). The geometric position is the same and the user receiver clock error contains the whole bias. The receiver assesses the common bias as an own clock offset.

This common component is not critical for the integrity monitoring because it does not affect the receiver position computation. Furthermore it can be easily recovered in the user clock error.

This technique then estimates the remaining component, that is

$$C\_differential = C\_proj + C\_reconstr$$

Besides it controls the common component observing the user clock estimation.

*E. Proposed Integrity Monitoring Technique: characteristics with respect to Least Squares Integrity Algorithm.*

The proposed approach uses the vector of residuals  $\overline{\Delta Y}$  on consecutive epochs and it assesses the different components of the error vector  $\overline{E}$ , reconstructing separately the  $C\_proj$  like the Least Squares Algorithm and then estimating the  $C\_reconstr$ .

Comparing this algorithm with the Least Squares Algorithm the following characteristics and advantages can be pointed out:

1. It detects **multiple failures**. There are no limitations on the number of  $\overline{E}$  components affected by biases, which can be detected. This means that it detects failures, which are discarded by oneshot techniques, in particular all those belonging to the subspace orthogonal to  $Null\overline{H}'$  and to  $(-1, -1, \dots, -1)_N$ .
2. It aims to reconstruct each single component of the vector  $\overline{E}(k)$ , through the monitoring of each single line of sight.
3. It monitors not only the norm of the residuals vector, but it uses the residuals to reconstruct the error vector  $\overline{E}(k)$  itself. From this estimation, the quantity  $\|\overline{E}(k)\|$  is also monitored. Simulations show the improvement in the failure detection capability especially in multiple failure cases.
4. The rigorous and mathematical relationship between  $\|\overline{E}(k)\|$  and  $\|\overline{Y}(k)\|$  has been used before the introduction of a statistical approach.
5. This technique provides also identification capability. In particular
  - a. The identification works with a number of satellites equal to the number of unknowns plus one (not plus 2 like RAIM identification technique).
  - b. Once failures have been detected, it provides an estimation of the  $\overline{E}(k)$  vector that is for each line of sight an estimation of the bias.

The following sections report simulation results to show the performance of the proposed approach.

## V. SIMULATION RESULTS

The first part of the simulations have been performed with the hypothesis that the error vector  $\overline{E}(k)$  does not change significantly on the consecutive epochs collected by the algorithm to provide each single output.

It is important to highlight that performance in terms of estimation error obtained with this hypothesis (described in the following section) do not reflect results obtained up to now with real data. This was an intermediate step of the performed analysis and simulations performed in these conditions are also described because they allow to show Least Squares Algorithm limitations and to explain the added value of the proposed technique with respect to these limitations.

Then this hypothesis is removed and in the next section the algorithm performances in real cases are presented.

The algorithm is still sensitive to error variation due to bad conditioning of the equations. This leads to a magnitude of the minimum detectable error in the order of hundred meters.

Further investigations on methodologies to solve bad conditioned problems are ongoing in order to improve the algorithm performances and reduce the magnitude of the minimum detectable error.

*Error assumed constant on 3 epochs.*

Real GPS Ephemeris data have been collected every second by a static receiver. A sub period of data with 6 satellites in view has been selected (duration 200s). The delay introduced by the proposed algorithm is then 2 epochs.

For each line of sight a random noise with 0.8m standard deviation has been added to each geometrical range. In addition to this, different combination of biases jump have been inserted at  $t=100s$ . In particular the following cases have been considered.

### A. Common component

This case focuses on the analysis of the Least Squares Integrity Algorithm with respect to the  $C\_common$ . In particular it is shown that this algorithm is not able to detect error vector, whose angle with respect to the subspace  $(-1, -1, \dots, -1)_N$  is small.

It is inserted the following bias

$$bias = [10, 10, 10, 10, 10, 15]$$

as shown in the Fig. 7.

The Least Squares Algorithm test statistic is not affected by the failure because the projection of the applied bias on the  $Null\bar{H}'$  is very small (Fig. 8). This case corresponds to a Missed Detection event. The Least Squares Algorithm performance in case of multiple failures depends on the angle between the error vector  $\bar{E}$  and the  $Null\bar{H}'$ .

The proposed algorithm tries to reconstruct the

$$C\_differential = [-0.833, -0.833, -0.833, -0.833, -0.833, 4.1667]$$

Results are shown in the Fig. 9.

The proposed algorithm is able to detect this critical failure conditions recovering the  $C\_differential$ . In particular the test statistic obtained estimating this component detects the presence of a bias, as shown in Fig. 9.

Once the algorithm detects a bias in this component, it uses the  $C\_differential$  to estimate also the  $C\_common$  and reconstructs the error vector  $\bar{E}$ , as shown in Fig. 10.

This further step corresponds to the identification capability of the algorithm.

### B. Single failure case

In this case the performance of the Least Squares Algorithm is discussed in presence of a single failure.

Actually the Least Squares Algorithm assumes to work only in this condition.

It is shown that there are cases in which the impact of the biases on the test statistic could be negligible and the detection capability of Least Squares Integrity Algorithm not sufficient to protect the user even in single failure cases.

It is considered the following bias:

$$bias = [0, 0, 0, 0, 0, 15]$$

$$C\_differential = [-2.5, -2.5, -2.5, -2.5, -2.5, 12.5]$$

Fig. 11, Fig. 12, Fig. 13 and Fig. 14 show the results and in particular the improved performances of the new algorithm

both in terms of detection and identification capability in single failure condition.

In fact Fig. 12 shows that the test statistic of Least Squares Algorithm is affected but by a small contribution. This is due to the fact that the detectability depends on the geometric configuration. This example shows a possible missed detection event of RAIM in case of single failure.

Finally it is observed that analogous results are valid also in cases of double failure.

### Multiple failures case

This section addresses multiple failure cases. In particular with a number of failures bigger than 2.

As reported in [2], with more than 2 failures there are cases in which the impact of the error vector on the Least Squares test statistic is null. These error vectors in fact belong to the  $Im\bar{H}$  and their projections on the  $Null\bar{H}'$  is zero.

In particular for each case of  $p$  failure it has been considered a bias vector obtained from the linear combination of the column of  $Im\bar{H}$  and having only  $p$  components different from 0.

In this paper only results on the cases with 3 and 6 failures have been inserted.

Following figures show the performance of the Least Squares Algorithm with respect to the proposed one, which reconstructs the  $C\_differential$ .

### C. 3 failures case

$$bias = [-8.068, -6.520, -1.921, 0, 0, 0]$$

$$C\_differential = [-5.316, -3.768, 0.830, 2.751, 2.751, 2.751]$$

### D. 6 failures case

$$bias = [6.594, 5.377, -3.035, -6.287, -6.746, -9.982]$$

$$C\_differential = [8.940, 7.723, -0.688, -3.940, -4.399, -7.635]$$

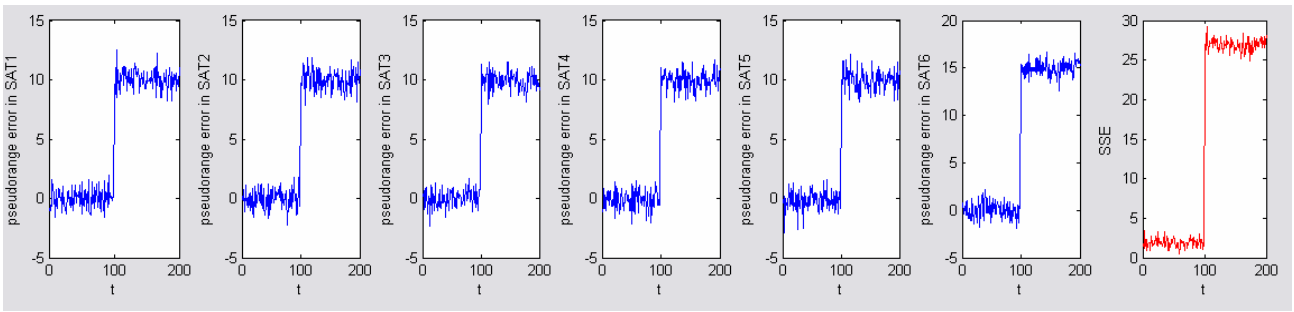


Fig. 7. Errors for each line of sight and Sum Squared Error in case of  $bias = [10,10,10,10,10,15]$ .

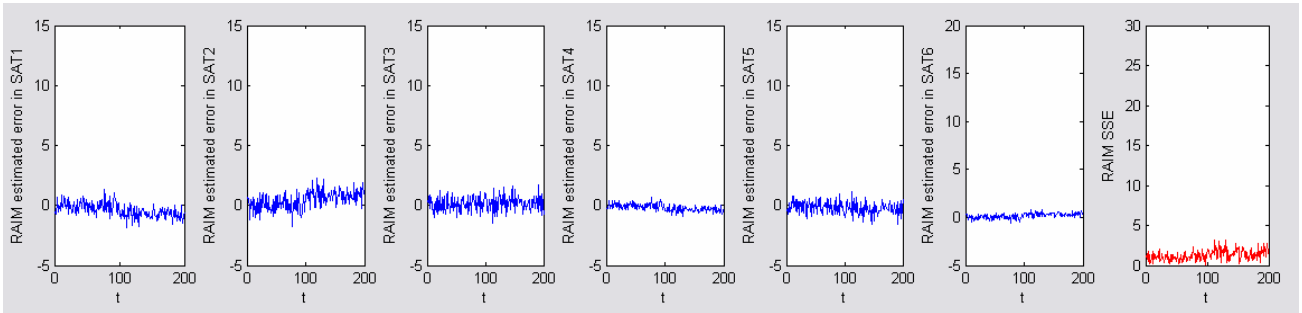


Fig. 8. Residual for each line of sight and RAIM Sum Squared Error (test statistic) in case of  $bias = [10,10,10,10,10,15]$ .

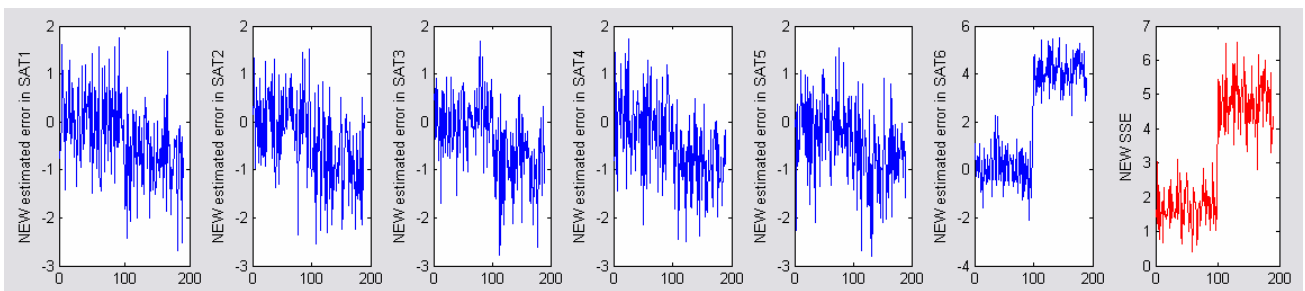


Fig. 9. New approach: Estimated  $C\_differential$  in case of  $bias = [10,10,10,10,10,15]$  (with Hyp of error constant on 3s).

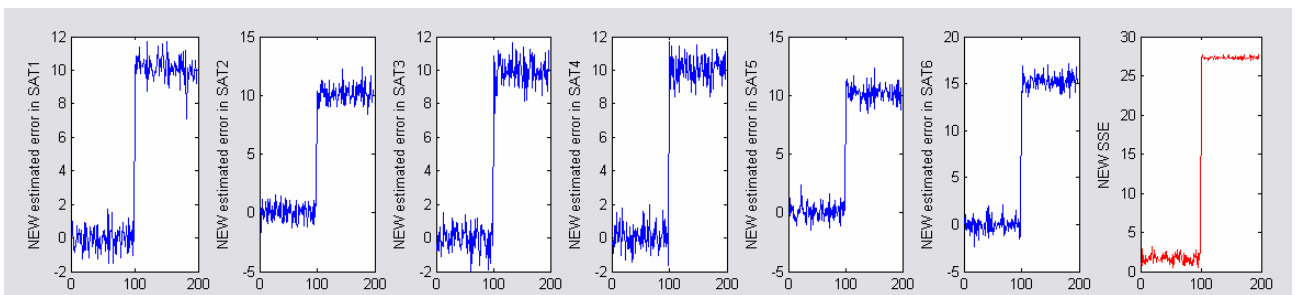


Fig. 10. New approach: Estimated Errors for each line of sight and Estimated Sum Squared Error in case of  $bias = [10,10,10,10,10,15]$  (with Hyp of error constant on 3s).

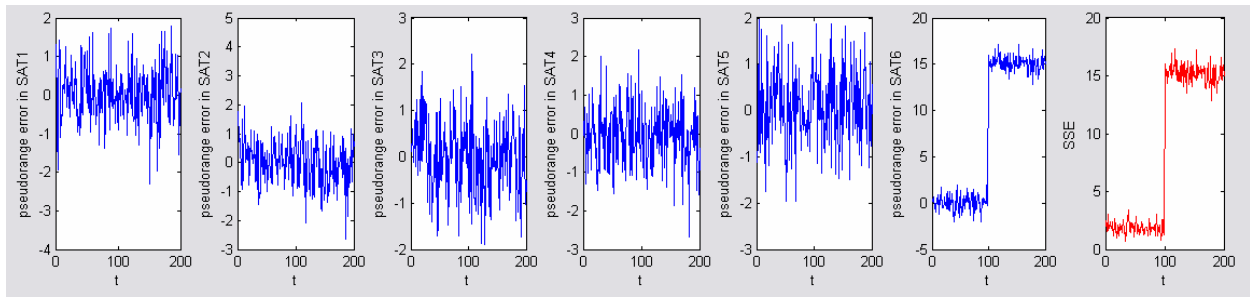


Fig. 11. Errors for each line of sight and Sum Squared Error in case of  $bias = [0, 0, 0, 0, 0, 15]$ .

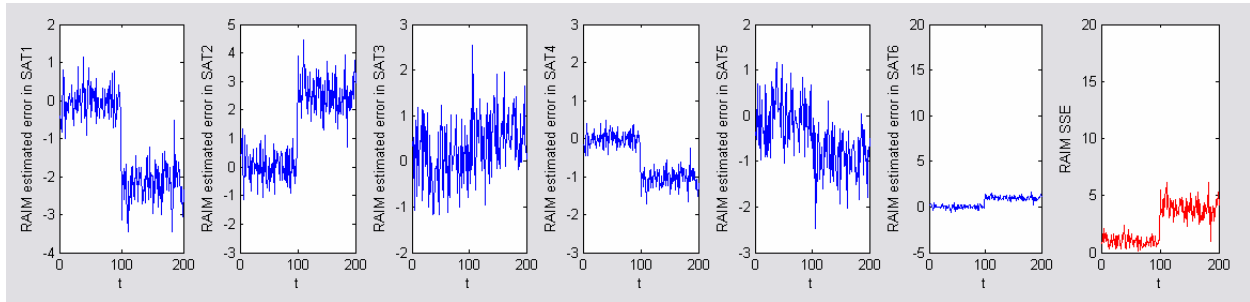


Fig. 12. Residual for each line of sight and RAIM Sum Squared Error (test statistic) in case of  $bias = [0, 0, 0, 0, 0, 15]$ .

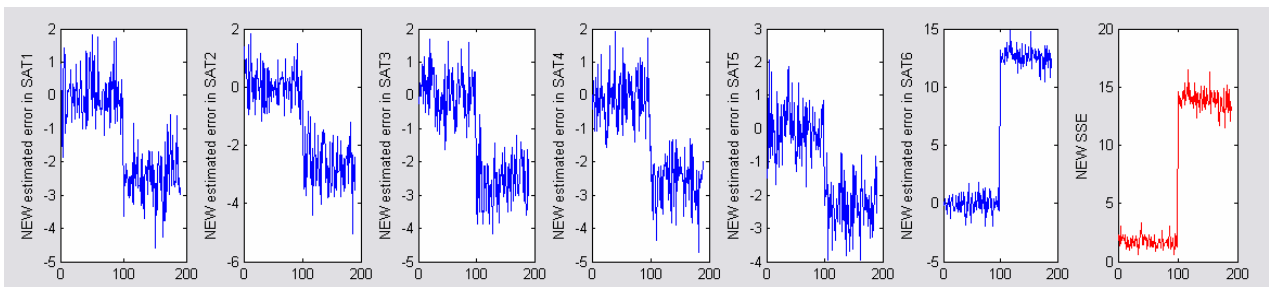


Fig. 13. New approach: Estimated  $C\_differential$  in case of  $bias = [0, 0, 0, 0, 0, 15]$  (with Hyp of error constant on 3s).

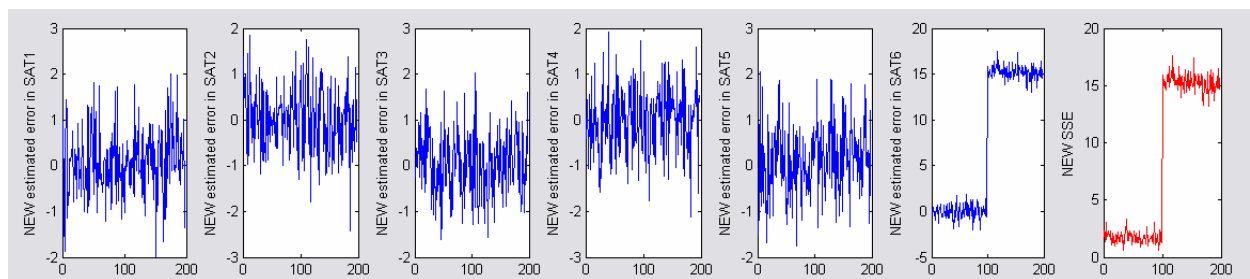


Fig. 14. New approach: Estimated Errors for each line of sight and Estimated Sum Squared Error in case of  $bias = [0, 0, 0, 0, 0, 15]$  (with Hyp of error constant on 3s).

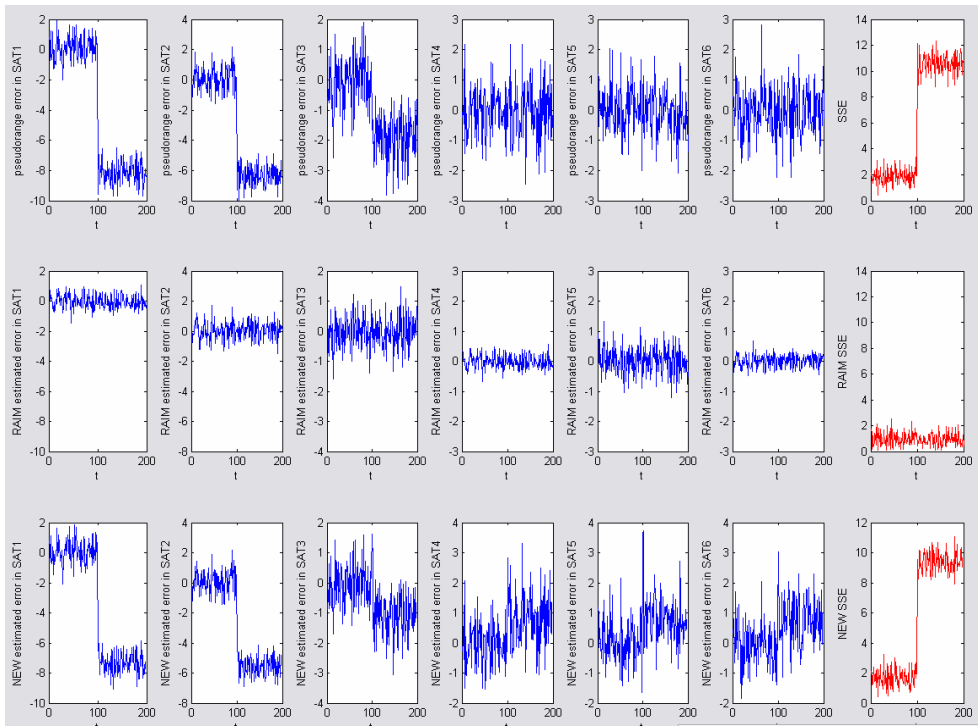


Fig. 15. Line of sight errors and SSE (1<sup>st</sup> row), residuals and Least Squares test statistic (2<sup>nd</sup> row), Estimated  $C\_differential$  (3<sup>rd</sup> row), in case of  $bias = [-8.068, -6.520, -1.921, 0, 0, 0]$  (with Hyp of error constant on 3s).

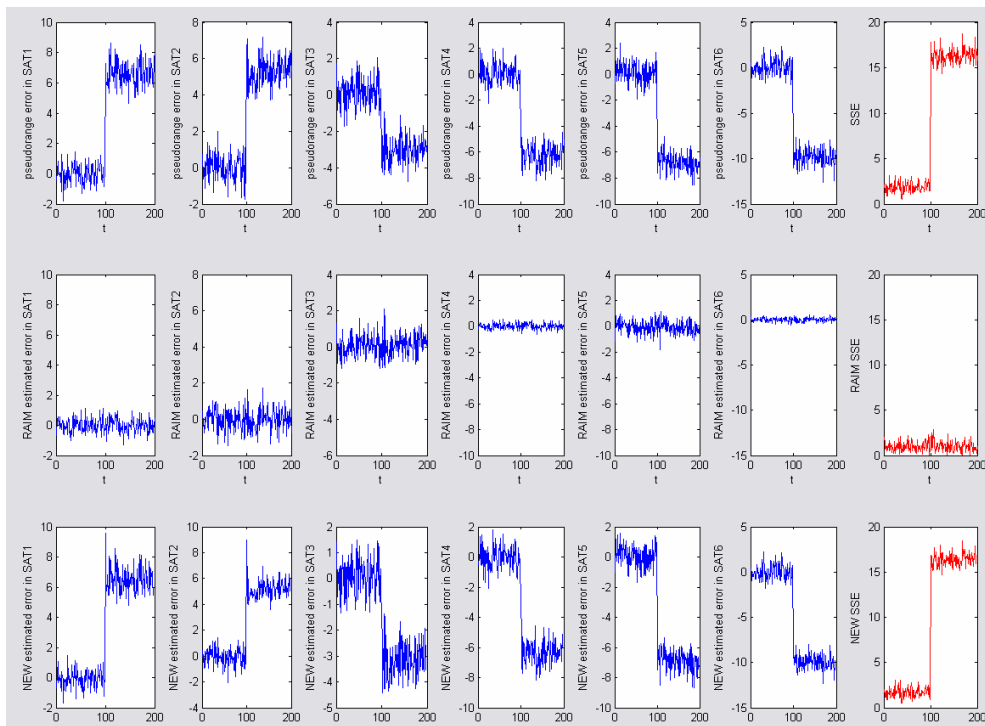


Fig. 16. Line of sight errors and SSE (1<sup>st</sup> row), residuals and Least Squares test statistic (2<sup>nd</sup> row), Estimated  $C\_differential$  (3<sup>rd</sup> row), in case of  $bias = [6.594, 5.377, -3.035, -6.287, -6.746, -9.982]$  (with Hyp of error constant on 3s).

## VI. PERFORMANCE REMOVING THE HYPOTHESIS OF CONSTANT ERROR

This section shows the performance of the proposed approach removing the hypothesis that the error remains constant on 3 epochs.

Simulation with real data show an increased sensibility of the algorithm with respect to error changes during the three epochs collected for each estimation. This is due to the bad conditioning of the problem.

More sophisticated methodologies are under investigation to solve the conditioning of the problem and improving the algorithm performance in particular to reduce the minimum errors detectable by the proposed algorithm.

In fact at the present, as it is shown in this section, results as described in the previous section are obtained but for biases with magnitude in the range of hundred of meters.

Furthermore it is also highlighted that up to now simulations with static users have been performed.

The extension to dynamic users is still under study: it will be investigated after the application of an improved technique for the conditioning of the problem.

Real data collected during GSTB-V1 by a Galileo Sensor Station every second has been used in these simulations.

A sub period with 6 satellites in view has been considered (duration 107s). The delay introduced by the algorithm is 2s.

It is observed the graphs in the first row of Fig. 17 and followings contain not the error in each pseudorange like the previous cases. In fact the user clock error observed in real data was in the order of  $10^5$ . Then in order to facilitate the visualization of the estimation capability, it has been necessary to remove the common component. Then the graphs display the  $C\_differential$  of the error for each line of sight.

Results relative to the following failure cases are shown to present algorithm performance of the proposed technique with respect to the Least Squares Algorithm in a real scenario.

### A. Single failure case

$$bias = [0, 0, 0, 0, 0, 900]$$

$$C\_differential = [-150, -150, -150, -150, -150, 750]$$

$$C\_common = (-305.450)[-1, -1, -1, -1, -1, -1]$$

### B. 2 failure case

$$bias = [-800, 0, 800, 0, 0, 0]$$

$$C\_differential = bias$$

### C. 3 failures case

$$bias = [90.222, -341.453, -532.850, 0, 0, 0]$$

$$C\_differential = [220.902, 210.773, -402.170, 130.680, 130.680, 130.680]$$

$$C\_common = (130.68)[-1, -1, -1, -1, -1, -1]$$

### D. 4 failures case

$$bias = [-174.086, 276.417, 853.938, 882.434, 0, 0]$$

$$C\_differential = [-480.537, -30.032, 547.488, 575.983, -306.450, -306.450]$$

$$C\_common = (-306.450)[-1, -1, -1, -1, -1, -1]$$

### E. 5 failures case

$$bias = [-168.122, 196.212, 710.031, 739.298, -28.195, 0]$$

$$C\_differential = [-409.659, -45.325, 468.493, 497.761, -269.732, -241.537]$$

$$C\_common = (-321.262)[-1, -1, -1, -1, -1, -1]$$

### F. 6 failures case

$$bias = [-699.645, -55.148, 177.503, -47.435, -703.666, -243.109]$$

$$C\_differential = [-354.395, -209.898, 522.754, 297.814, -358.416, 102.141]$$

$$C\_common = (354.250)[-1, -1, -1, -1, -1, -1]$$

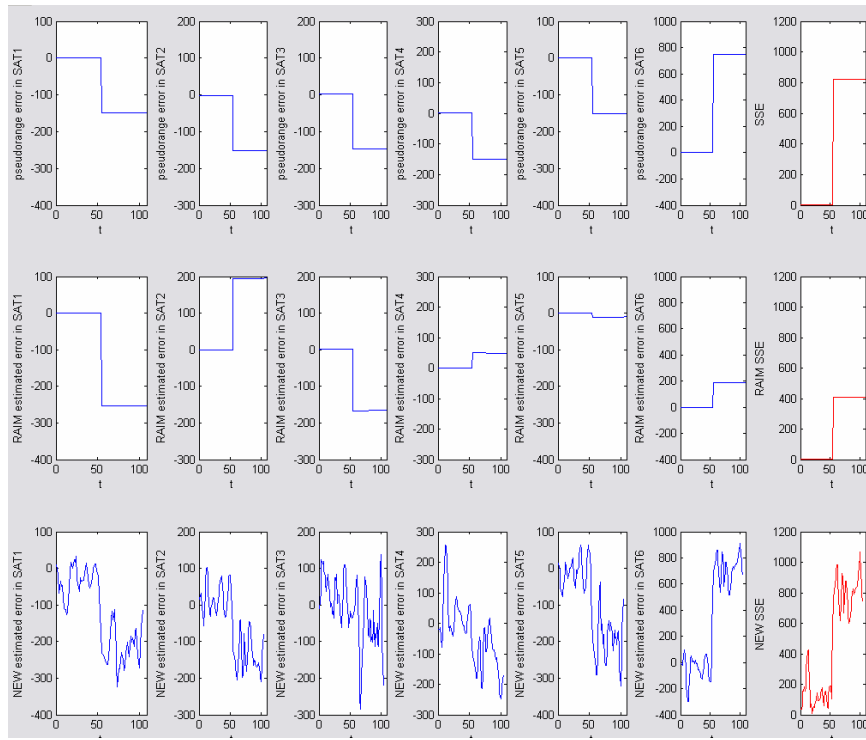


Fig. 17.  $C\_differential$  and norm of  $C\_differential$  (1<sup>st</sup> row), residuals and Least Squares test statistic (2<sup>nd</sup> row), Estimated  $C\_differential$  (3<sup>rd</sup> row) with  $bias = [0, 0, 0, 0, 0, 900]$ .

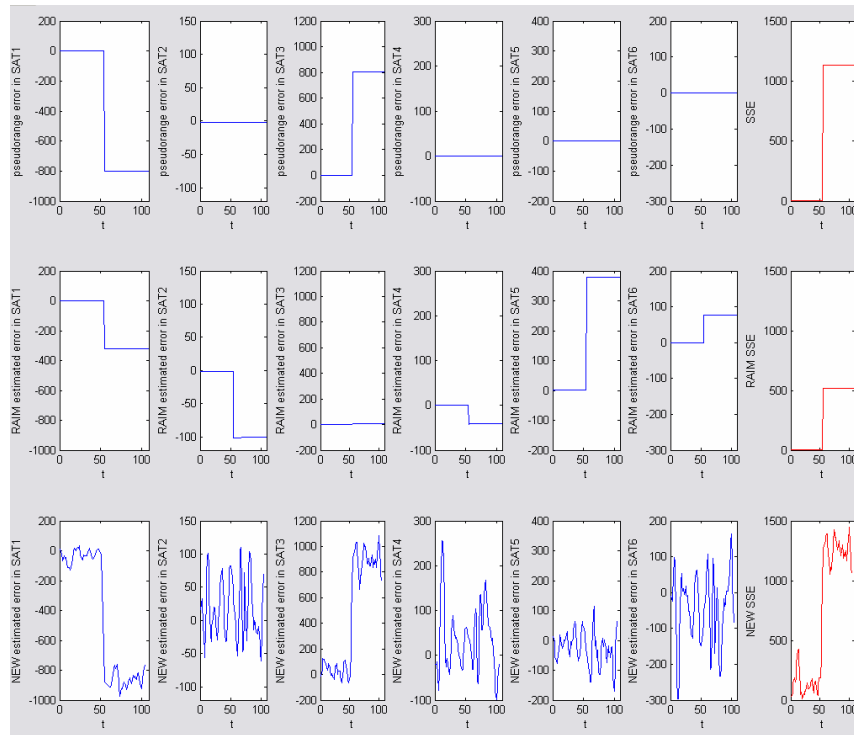


Fig. 18.  $C\_differential$  and norm of  $C\_differential$  (1<sup>st</sup> row), residuals and Least Squares test statistic (2<sup>nd</sup> row), Estimated  $C\_differential$  (3<sup>rd</sup> row) with  $bias = [-800, 0, 800, 0, 0, 0]$ .

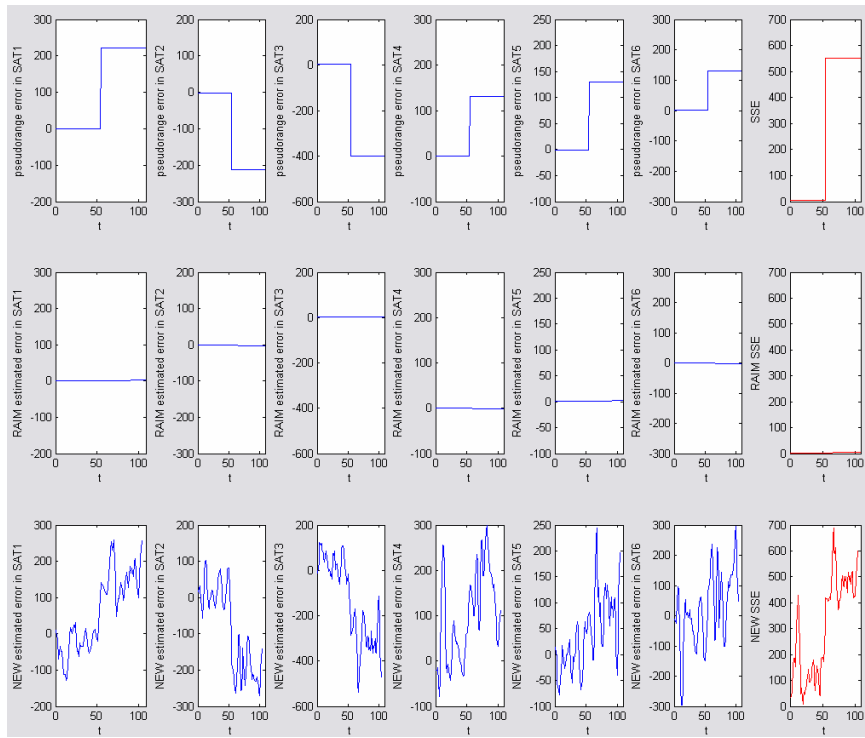


Fig. 19.  $C\_differential$  and norm of  $C\_differential$  (1<sup>st</sup> row), residuals and Least Squares test statistic (2<sup>nd</sup> row), Estimated  $C\_differential$  (3<sup>rd</sup> row) with  $bias = [90.222, -341.453, -532.850, 0, 0, 0]$ .

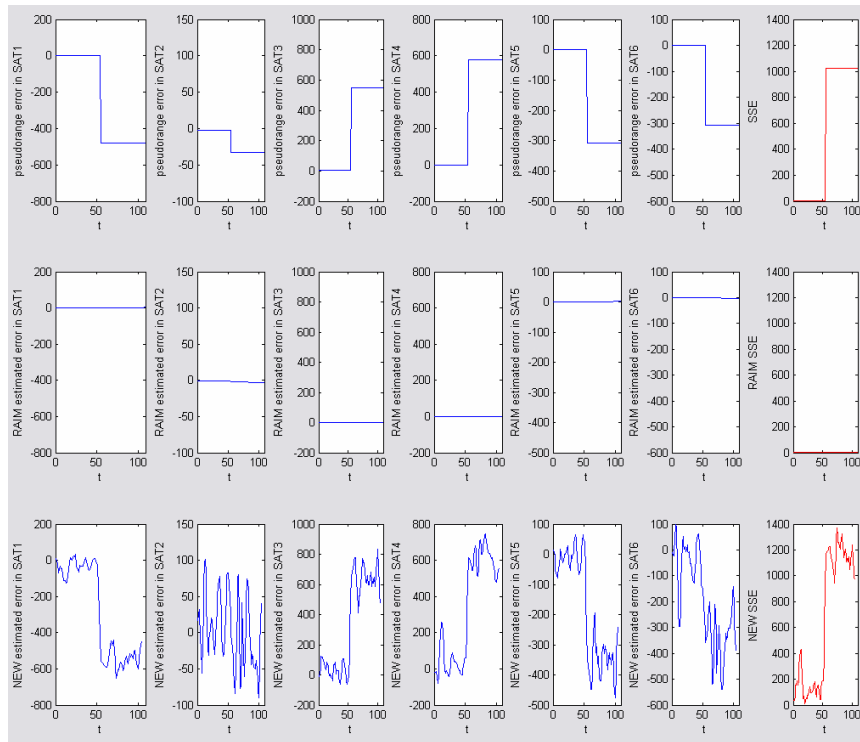


Fig. 20.  $C\_differential$  and norm of  $C\_differential$  (1<sup>st</sup> row), residuals and Least Squares test statistic (2<sup>nd</sup> row) Estimated  $C\_differential$  (3<sup>rd</sup> row) with  $bias = [-174.086, 276.417, 853.938, 882.434, 0, 0]$ .

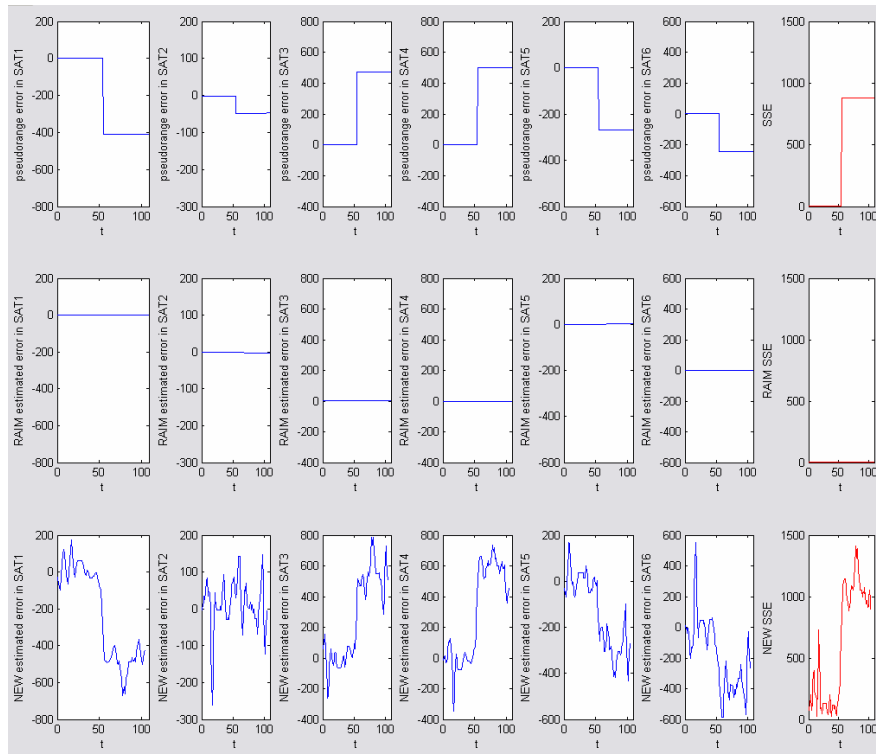


Fig. 21.  $C\_differential$  and norm of  $C\_differential$  (1<sup>st</sup> row), residuals and Least Squares test statistic (2<sup>nd</sup> row), Estimated  $C\_differential$  (3<sup>rd</sup> row) with  $bias = [-168.122, 196.212, 710.031, 739.298, -28.195, 0]$ .

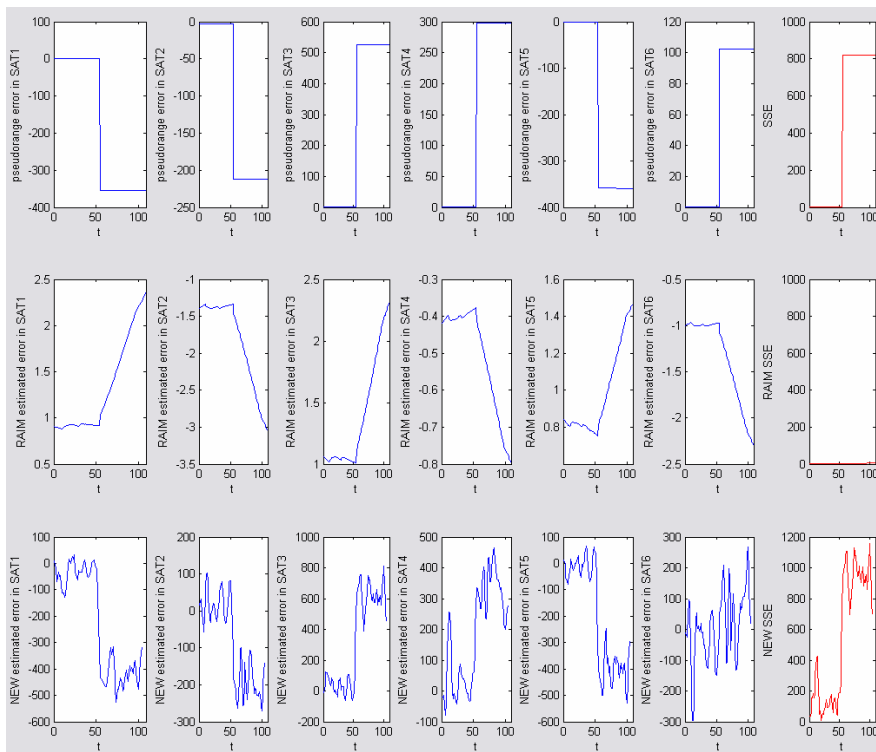


Fig. 22.  $C\_differential$  and norm of  $C\_differential$  (1<sup>st</sup> row), residuals and Least Squares test statistic (2<sup>nd</sup> row), Estimated  $C\_differential$  (3<sup>rd</sup> row) with  $bias = [-699.645, -55.148, 177.503, -47.435, -703.666, -243.109]$ .

## VII. CONCLUSIONS AND WAY FORWARD

Integrity Monitoring at receiver level plays an essential role in protecting the user receiver against failure conditions, in particular with the aim of certifying a GNSS system as stand alone positioning mean in aeronautical applications. In fact it is the only technique able to protect the user against local errors.

Least Squares Integrity Algorithm presents limitations, which are described in the paper. In particular it is able to protect only against single failure. But to fulfill the small integrity risk probabilities required by the aeronautical operations, multiple failure events cannot be discarded by the receiver.

This paper presents an investigation on the Least Squares Algorithm and proposes a new algorithm able to overcome its limitations.

In particular the proposed technique is able to detect any multiple failure condition through the elaboration of consecutive epochs. Furthermore it aims to estimate the error affecting each line of sight, enabling the identification capability and the possibility to assess the error magnitude.

This approach introduces a delay of around 2s (it is maximum 3s in a GNSS only receiver and 5s in a combined GPS/Galileo receiver). This limited (and deterministic) delay is not critic because it allows meeting the aeronautical requirement of time to alarm smaller than 6s.

Simulation results performed with static user cases, and described in the paper, show the improved detectability performance of the proposed algorithm with respect to Least Squares Algorithm, either in single failure condition and above all in different multiple failure conditions. Besides the algorithm provides an estimation of the error affecting each line of sight.

More sophisticated techniques to solve the ill-conditioning of the problem are under investigation in order to reduce the magnitude of the minimum detectable error and test the algorithm performances in aeronautical dynamic user cases.

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