A KALMAN FILTER FOR THE INTEGRATION OF A LOW COST INS AND AN ATTITUDE GPS

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BIOGRAPHY

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Robert Wolf received a diploma in aerospace engineering in 1995 from the Munich University of Technology. In 1995 he joined the Institute of Geodesy and Navigation as a Research Associate, working in the field of GPS/INS integration.

ABSTRACT

The availability of low cost inertial sensors results in increasing interest in integrated GPS/INS systems. Such an IMU, Systron Donner's MotionPak, has been purchased by the Institute of Geodesy and Navigation (IfEN) in order to develop an integrated GPS/INS attitude and position determination system. As GPS component for the integrated system, IfEN has selected the Trimble Advanced Navigation Sensor (TANS) Vector receiver system, which is a multi antenna attitude determination and position location system. IfEN has developed a real-time navigation software to calculate position, velocity and attitude from the outputs of the MotionPak gyroscopes and accelerometers. Since some computational advantage was found in using more simple inertial algorithms, most of our calculations are performed in the inertial reference frame and then transformed to the navigation frame. This software takes into account algorithmic errors like coning and sculling. The test results for the free drifting INS, obtained with the MotionPak navigation software, show errors in the range of ± 0.5° to ± 1° in attitude, in velocity of ± 10 m/s and in position of several hundreds of meters after the first five minutes starting from initial alignment. These relatively large errors, due to the low bias stability of the MotionPak sensors had been significantly improved by modelling the sensor errors and using a closed loop Kalman filter. The filter is updated by using GPS data from the TANS Vector. The Trimble TANS Vector provides, besides position and velocity, attitude data with an accuracy of 0.5° for pitch and roll and 0.3° for azimuth (1 m baseline). The differences between INS position, velocity or attitude and the corresponding GPS derived values are used as measurements to update the Kalman filter. Navigation and sensor errors are estimated and fed back to correct the navigation process and compensate the sensor errors. Due to the availability of up to eleven different measurement types a Kalman filter with 27 states could be used without any stability problems. The state vector contains the most important sensor errors like bias, scale factor error and temperature sensibility. The test results for the integrated system shows an attitude accuracy of up to 0.04° (1σ) for pitch and roll and up to 0.23° (1σ) for azimuth. The system obtained by this integration provides GPS like position accuracy (~10 cm using differential GPS) and relatively high attitude accuracy combined with high data output rate, thus being suitable for many applications even in a high dynamic environment.

HARDWARE COMPONENTS

The low cost integrated system uses the following hardware components:

– Systron Donner MotionPak, inertial sensor assembly

– Trimble TANS Vector, multi antenna GPS receiver

– Keithley DAS 1802 HR, 16 bit data acquisition plug in board

– Guide Technology GT401 event timer

– Packard Bell 486/ DX 66, personal computer

The output of the MotionPak is sampled at a rate of 600 Hz with 16 bit resolution. The TANS Vector is connected to the computer via serial link. Data transmission is performed at 38.4 kBaud.

SYNCHRONIZATION

The 1PPS output of the TANS Vector is used to trigger the GT401 timing board, which is therefore synchronized with GPS time. The INS Software is using it's own time base resulting from the number of acquired samples. INS time and GPS time are compared every 1000 samples, thus yielding actual time base offset and drift of the sampling clock, which is about 0.8 seconds per day.

GPS RECEIVER

The Trimble TANS Vector is multi antenna position and attitude determination system. It is equipped with four antennas, one master and three slave antennas and a six channel GPS receiver.

The three attitude angles roll, pitch and azimuth are determined by differential carrier phase measurements (see Figure 1) between the master antenna and each of the three slave antennae.

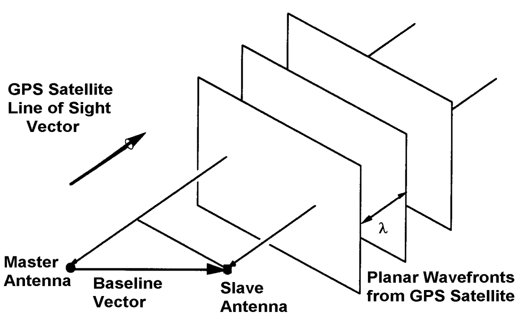

Figure 1: Differential Carrier Phase Measurement

The TANS Vector provides two serial data channels, assigned as channels A and B.

Channel B can be used to output all data, including attitude data at a rate up to 10 Hz. Position and velocity information has low priority and is therefore only measured every 1.5 seconds. Transmission of position and velocity can be delayed up to 2 seconds. This has to be taken into consideration in real time applications.

Channel A provides all data except attitude. Its main purpose is to serve as data input channel in differential mode (RTCM corrections). Transmission rate is fixed to 9600 Baud.

Table 1 indicates the specifications of the Trimble TANS Vector.

Azimuth accuracy	0.3 ° (RMS), 1 m Baseline
Position accuracy	100 m horizontal, 156 m vertical, SA enabled Differential GPS, Base station within 500km: 5m horizontal, 8 m vertical
Velocity accuracy	0.2 m/s , without SA Differential GPS: 0.1 m/s
Power	28 VDC , 7 Watts
Weight	Receiver: 1.42 kg Antenna: 0.19 kg
Dimensions	Receiver: 127 x 241 x 64 mm^3 Antenna: 96 x 102 x 13 mm^3

Table 1: Specifications of the TANS Vector

Trimble delivers the TANS Vector completely integrated with receiver and processor unit and antennas in a 0.52 x 0.52 x 0.08 m^3 housing, thus getting 0.58 m baseline on the diagonal. Figure 2 indicates the reference axis for the four antenna system.

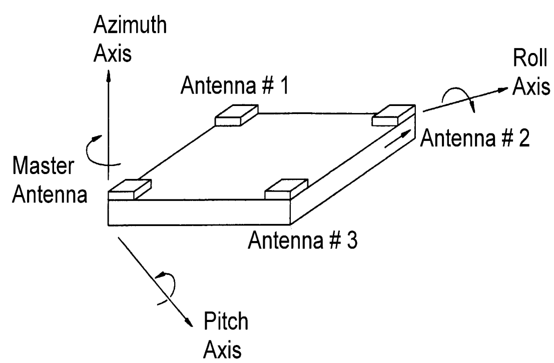

Figure 2: Attitude Reference Axis Definition

INERTIAL MEASUREMENT UNIT

The Systron Donner MotionPak is a six degree of freedom inertial sensor, measuring three linear accelerations along orthogonal axes and three angular rates. The angular rates are sensed using oscillating quartz tuning forks and linear accelerations are sensed using a vibrating quartz beam. Additionally, it has a temperature sensor. This is necessary, because the MotionPak is not temperature compensated. The output of the MotionPak is a voltage proportional to acceleration or angular rate input. Table 2 gives an overview of the specifications of the MotionPak.

Size	7.6 x 7.6 x 9.0 cm³
Weight	0.81 kg
Power	
Input Voltage	+/- 15 VDC
Input Power	7 Watts
Measurement Range	
Gyro	± 200 °/s
Accelerometer	± 5 g
Inertial Sensor Performance	
Gyro	
Null (rate sensor bias)	0.09 °/s
Scale factor error	< 1%
Bias Temperature Change	< ± 3 °/s
Scale Factor Temperature Change	± 0.03% /°C
Noise	0,01 °/s √Hz
Misalignment	< 0.33 °
Accelerometer	
Bias	± 3.6 mg
Scale factor error	< 1%
Bias Temperature Change	± 15 µg
Scale Factor Temperature Change	± 0.001 %/°C
Noise	< 2.5 mV RMS
Misalignment	< 0.2 °

Table 2: Specification of the MotionPak

INERTIAL NAVIGATION SOFTWARE

The output of the MotionPak contains three accelerations and three angular rates referenced to sensor coordinates. These values have to be integrated to get velocity, position and attitude.

There are several possibilities of mechanization of the strapdown algorithms. We found some computational advantages in computing the direction cosine matrix and integrating the specific forces in an earth centered inertial frame. From there, the desired values can easily be transformed into the desired reference frame, normally the local level navigation frame with north, east and down axes.

To avoid algorithmic errors like coning and sculling resulting form the direction cosine matrix being not constant during the sampling interval, the inertial data sampled at 600 Hz is corrected by pre-integration. These delta-velocity and delta-angle values are processed now at a rate of 200 Hz.

In real time mode, this part of the software has the highest priority, because the values are computed by iteration. If for example some angular rate information is lost while turning the sensor (or the host vehicle), the attitude angle will be too small. This can be compared with platform tilt of an mechanical INS.

High accuracy INS can determine the initial transformation matrix from the sensor output. Due to large and unstable gyro drifts, the MotionPak can only determine the "down" direction from the accelerometer output, i.e. pitch and roll angle. In the integrated system, the azimuth is taken from the TANS Vector.

KALMAN FILTER DESIGN

The Kalman filter is used to estimate and correct the errors of the INS. Integrating an INS with the a multi antenna system like the TANS Vector means that all navigation states can be observed. This allows a filter design using a large state vector.

The normal way is to start with a basic filter design modeling only the dynamics of the navigation errors. Such a filter concept has been developed e.g. by Schmidt (1978). This basic design is then augmented with additional states resulting from the error analysis of the sensors.

The used filter design considers the major error sources of the low cost INS and their impact on the navigation errors. The decision, whether an error source is modeled as state or only considered as "noise" being added to acceleration or angular rate, is based upon the analysis of error behavior represented by the values in table 2.

First let us consider the gyro drift or rate sensor bias. This value is of the magnitude of 0.1 °/s and is defined as a day to day error. This very large value has to be estimated in the Kalman filter. The accelerometer bias is also relatively large. These shall be our first 6 states.

The scale factor errors of the gyros and accelerometers can be up to 1%, which is a very large value. Thus, they will be also estimated. Considering the scale factor error for 3 gyros and 3 accelerometers add 6 further states.

Temperature change of gyro bias can be up to 3 °/s over the whole temperature range of –40 °C to 85 °C. This error source is also modeled as a state. The accelerometers show a much more stable temperature performance, so their error contribution can be considered as noise.

Besides the INS error states three further states are used to deal with a problem, introduced when integrating two attitude sensors.

STATE VECTOR

The state vector of the filter consists of the following 27 elements:

$$\vec{\mathbf{X}}_{INS/GPS} = \left[\boldsymbol{\delta\Theta}, \boldsymbol{\delta}\mathbf{v}, \boldsymbol{\delta}\mathbf{r}, \mathbf{d}, \mathbf{b}, \mathbf{d_T}, \boldsymbol{\kappa}_\mathbf{a}, \boldsymbol{\kappa}_\mathbf{g}, \boldsymbol{\Psi}\right]$$

The first nine states represent the INS navigation errors:

$\boldsymbol{\delta\Theta}$	Vector of attitude error with respect to north, east and down axes
$\boldsymbol{\delta}\mathbf{v}$	Vector of velocity error in latitude rate, longitude rate and altitude rate
$\boldsymbol{\delta}\mathbf{r}$	Vector of position error in latitude, longitude and altitude

The next 15 states result from the error analysis and express the major INS sensor error sources:

$\mathbf{d}$	Vector of uncompensated gyro drift
$\mathbf{b}$	Vector of uncompensated accelerometer bias
$\mathbf{d_T}$	Vector of gyro drift due to temperature changes
$\boldsymbol{\kappa}_\mathbf{g}$	Vector of gyro scale factor errors
$\boldsymbol{\kappa}_\mathbf{a}$	Vector of accelerometer scale factor errors

The next three states are used to compensate the angle offsets between the coordinate systems off the INS and the multi antenna GPS. They represent three Euler angles.

$\boldsymbol{\Psi}$	3 offset angles between GPS and INS axes

The state estimates are fed back to correct the navigation computation and the sensor output. The estimation of the offset angles is necessary because the TANS Vector shows a (nearly) constant offset in attitude that can't be neglected.

DYNAMIC COUPLING AND STATE TRANSITION

Dynamic coupling between states is expressed by the dynamic matrix F with white process noise input:

$$\dot{\vec{\mathbf{x}}} = \mathbf{F} \cdot \vec{\mathbf{x}} + \vec{\mathbf{w}}$$

The transition matrix, necessary for prediction of states and covariance is obtained by

$$\boldsymbol{\Phi} \approx \mathbf{I} + \mathbf{F} \cdot \Delta t + \frac{1}{2}\mathbf{F}^2 \cdot \Delta t^2$$

The dynamic matrix $\mathbf{F}$ is shown below, with brief explanation of the submatrices following:

$$\mathbf{F} = \begin{bmatrix} \multicolumn{3}{c}{} & \mathbf{C_b^n} & \bar{0} & \mathbf{C_b^n} \cdot \Delta T & \mathbf{C_b^n \Omega^b} & \bar{0} & \bar{0} \\ \multicolumn{3}{c}{\mathbf{F}_{Free}\ [9x9]} & \bar{0} & \mathbf{DC_b^n} & \bar{0} & \bar{0} & \mathbf{DC_b^n F^b} & \bar{0} \\ \multicolumn{3}{c}{} & \bar{0} & \bar{0} & \bar{0} & \bar{0} & \bar{0} & \bar{0} \\ \bar{0} & \bar{0} & \bar{0} & -\mathbf{I} \cdot \beta_g & \bar{0} & \bar{0} & \bar{0} & \bar{0} & \bar{0} \\ \bar{0} & \bar{0} & \bar{0} & \bar{0} & -\mathbf{I} \cdot \beta_a & \bar{0} & \bar{0} & \bar{0} & \bar{0} \\ \bar{0} & \bar{0} & \bar{0} & \bar{0} & \bar{0} & \bar{0} & \bar{0} & \bar{0} & \bar{0} \\ \bar{0} & \bar{0} & \bar{0} & \bar{0} & \bar{0} & \bar{0} & \bar{0} & \bar{0} & \bar{0} \\ \bar{0} & \bar{0} & \bar{0} & \bar{0} & \bar{0} & \bar{0} & \bar{0} & \bar{0} & \bar{0} \\ \bar{0} & \bar{0} & \bar{0} & \bar{0} & \bar{0} & \bar{0} & \bar{0} & \bar{0} & \bar{0} \end{bmatrix}$$

The submatrix $\mathbf{F}_{free}$ is taken from Schmidt (1978) and is not shown in detail here.

ΔT	Difference between temperature output and last calibration temperature
β	Correlation coefficients of gyros (index g) and accelerometers (index a)
$\mathbf{I}$	3x3 unit matrix

The $\mathbf{C_b^n}$ Matrix is the transformation matrix between body and navigation frame and is defined as

$$\mathbf{C_b^n} = \begin{bmatrix} \cos\gamma \cdot \cos\beta & \begin{matrix} \cos\gamma \cdot \sin\beta \cdot \sin\alpha \\ -\sin\gamma \cdot \cos\alpha \end{matrix} & \begin{matrix} \cos\alpha \cdot \sin\beta \cdot \cos\gamma \\ +\sin\gamma \cdot \sin\alpha \end{matrix} \\ \cos\beta \cdot \sin\gamma & \begin{matrix} \sin\gamma \cdot \sin\beta \cdot \sin\alpha \\ +\cos\gamma \cdot \cos\alpha \end{matrix} & \begin{matrix} \cos\alpha \cdot \sin\beta \cdot \sin\gamma \\ -\sin\alpha \cdot \cos\gamma \end{matrix} \\ -\sin\beta & \cos\beta \sin\alpha & \cos\alpha \cdot \cos\beta \end{bmatrix}$$

with

α	Roll
β	Pitch
γ	Azimuth.

This matrix is needed to relate sensor errors, normally defined in body coordinates to the navigation errors defined in local horizon frame.

The scaling matrix $\mathbf{D}$ is defined by

$$\mathbf{D} = \begin{bmatrix} \frac{1}{R} & 0 & 0 \\ 0 & \frac{1}{R\cos\Phi} & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

and is necessary to transform linear velocity to latitude or longitude rate.

$\mathbf{F^b}$ and $\boldsymbol{\Omega}^\mathbf{b}$ are diagonal matrices containing the gyros and accelerometers output.

$$\mathbf{\Omega^b} = \begin{bmatrix} \omega_x^b & 0 & 0 \\ 0 & \omega_y^b & 0 \\ 0 & 0 & \omega_z^b \end{bmatrix}$$

$$\mathbf{F^b} = \begin{bmatrix} f_x^b & 0 & 0 \\ 0 & f_y^b & 0 \\ 0 & 0 & f_z^b \end{bmatrix}$$

With respect to process noise $\vec{\mathbf{w}}$ we add white noise to each state in order to keep the Kalman filter adaptive.

OBSERVATIONS

Eleven different observations are used to update the Kalman filter:

Three dimensional position
Three dimensional velocity
Three Euler angles
Two level angles

Position, velocity and the three Euler angles roll, pitch and azimuth are provided by the TANS Vector. The two level angles (pitch and roll), which are off course also Euler angles are provided by the accelerometers output during alignment. The observation equations are given in detail below.

GPS POSITION

The observation, used to update the Kalman filter is the difference between the INS and GPS computed position vectors, thus yielding the simple error equation

$$\vec{\mathbf{r}}_{INS} - \vec{\mathbf{r}}_{GPS} = \boldsymbol{\delta}\vec{\mathbf{r}} + \vec{\mathbf{n}}$$

with

$\vec{\mathbf{r}}$	position vector
$\boldsymbol{\delta}\vec{\mathbf{r}}$	vector of position error (states)
$\vec{\mathbf{n}}$	white noise.

GPS VELOCITY

The way to observe the velocity error is to compute the difference between INS and GPS derived velocity. The velocity difference has to be related to the velocity error state via scaling matrix **D**, described before. This equation is also simple, because velocity error can be observed directly.

$$\vec{\mathbf{v}}_{INS} - \vec{\mathbf{v}}_{GPS} = \mathbf{D}^{-1} \cdot \boldsymbol{\delta}\vec{\mathbf{v}} + \vec{\mathbf{n}}$$

GPS ATTITUDE

The error in the three Euler angles can be observed by measuring the difference between INS and GPS attitude.

The components of the state vector related to attitude are no Euler angles, but orientation errors around north, east and down axes. These can be thought of as the "tilt" of the computational platform in a strapdown application.

Assuming that the tilt angles are small, one may write

$$\delta\text{Roll} = \frac{\cos\gamma}{\cos\beta} \cdot \delta\Theta_N + \frac{\sin\gamma}{\cos\beta} \cdot \delta\Theta_E + \Psi_{Roll} + n$$

$$\delta\text{Pitch} = -\sin\gamma \cdot \delta\Theta_N + \cos\gamma \cdot \delta\Theta_E + \Psi_{Pitch} + n$$

$$\delta\text{Azimuth} = \frac{\sin\beta \cdot \cos\gamma}{\cos\beta} \cdot \delta\Theta_N + \frac{\sin\beta \cdot \sin\gamma}{\cos\beta} \cdot \delta\Theta_E + \delta\Theta_D + \Psi_{Azimuth} + n$$

with

$\delta\Theta_{N,E,D}$	Orientation errors with respect to north, east and down axes.
Ψ	Offset angle between INS and GPS axes.

LEVEL UPDATE FROM ACCELEROMETER OUTPUT

During initial alignment the Euler angles pitch and roll are determined from the accelerometers output using following equations:

$$\alpha = \arctan\left(\frac{f_y}{f_z}\right)$$

$$\beta = \arcsin(-f_x)$$

This procedure must be thought of as a course alignment, because an error is introduced by deriving the "down" direction from (biased) accelerometers. During filter alignment these observations may still be used if the error introduced by the accelerometers is modeled correctly. The following equations are easily derived from the equations for α and β by Taylor series expansion and linearization.

$$\delta\text{Roll} = \frac{\cos\gamma}{\cos\beta} \cdot \delta\Theta_N + \frac{\sin\gamma}{\cos\beta} \cdot \delta\Theta_E + \frac{f_z}{f_y^2 + f_y^2} \cdot b_y - \frac{f_y}{f_y^2 + f_y^2} \cdot b_z + n$$

$$\delta Pitch = -\sin\gamma \cdot \delta\Theta_N + \cos\gamma \cdot \delta\Theta_E + \frac{\sqrt{|f|^2 - f_x^2}}{|f|^2} \cdot b_x$$

$$- \frac{f_x \cdot f_y}{|f|^2 \cdot \sqrt{|f|^2 - f_x^2}} \cdot b_y - \frac{f_x \cdot f_z}{|f|^2 \cdot \sqrt{|f|^2 - f_x^2}} \cdot b_z + n$$

Note that these observations do not depend on the offset between INS and GPS coordinate frame.

KALMAN FILTER RESULTS

The following figures show the test results for a static test. The data has been obtained in post processing, using simulated differential GPS position and real attitude data from the TANS Vector to update the filter.

FILTERED POSITION

Using differential GPS with carrier phase ambiguity resolution yields an accuracy < 10 cm after the ambiguities could be fixed. The integrated system shows no improvement regarding accuracy. The INS serves mainly as an interpolator between GPS solutions. The next three figures show the position errors in north, east and vertical components.

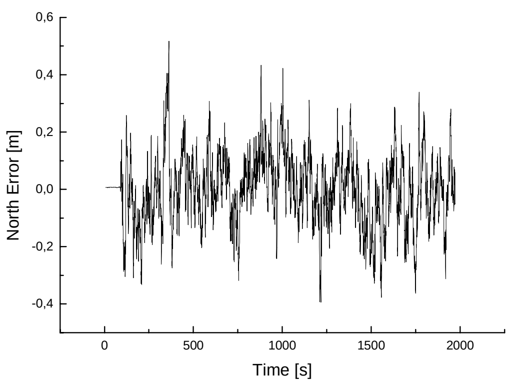

Figure 3: North error of filtered position

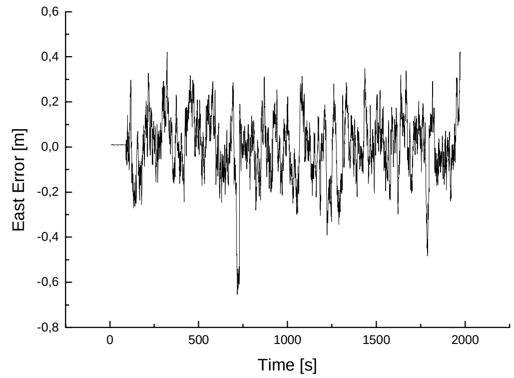

Figure 4: East error of filtered position

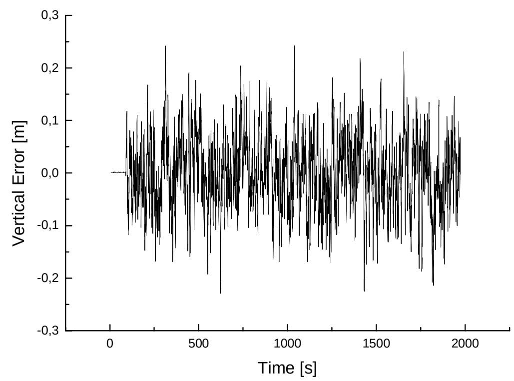

Figure 5: Vertical error of filtered position

The following table indicates the 1 σ position error for the north, east and down component.

North Error 1 σ [m]	0.1
East Error 1 σ [m]	0.1
Vertical Error 1 σ [m]	0.1

Table 3: Position **e**rror

FILTERED ATTITUDE

The main interest, when integrating INS and GPS lies in the accuracy of attitude angles. Especially in a high dynamic environment, the TANS Vectors data output rate of 10 Hz would be too low. Also the fact that the GPS data is transmitted with a delay could be a problem for real time applications. The next three figures show the attitude errors.

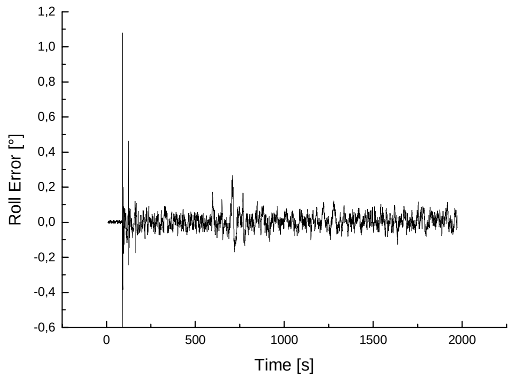

Figure 6: Roll error

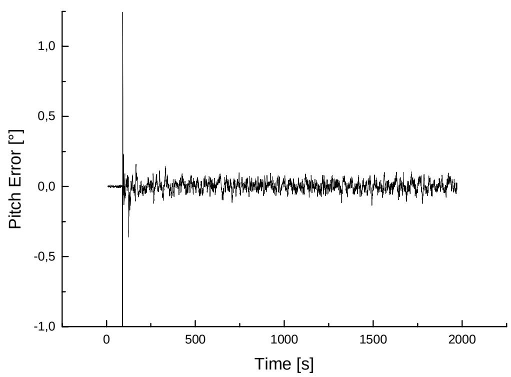

Figure 7: Pitch error

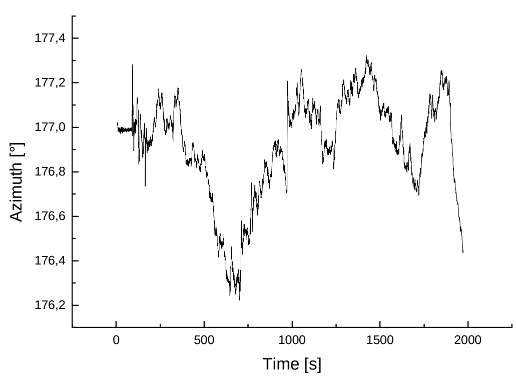

Figure 7: Azimuth

It can be seen that the roll and pitch angles show relatively high accuracy. The reason is the dynamic coupling between a tilt of the computational platform an the velocity error, resulting in a position error. By observing the position and velocity errors, the platform tilt can be estimated with relatively accuracy.

If the navigation system (i.e. the host vehicle) is moved, an error in azimuth also results in a position error, but the coupling is weak. Noticeable improvements in the estimation of the azimuth error are obtained only at higher velocities. For the static test, the accuracy of the integrated system depends on the accuracy of the azimuth provided by the TANS Vector. The following table indicates the achieved accuracy in roll, pitch and azimuth.

Roll Error 1 σ [°]	0.04
Pitch Error 1 σ [°]	0.04
Azimuth Error 1 σ [°]	0.23

Table 4: Attitude error

CONCLUSION

It has been shown that an integration of a low cost inertial sensor with a multi antenna GPS leads to relatively good accuracy in position, roll and pitch, if the errors of the IMU are modeled correctly in the Kalman filter. Further improvements in azimuth accuracy can be achieved by enlarging the baseline.

Although using 27 states, the filter still remains stable even if the number of observations is reduced. With today's computers the computational effort can be handled without problems even in real time.

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