Real-Time Station Clock Synchronization for GNSS Integrity Monitoring

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BIOGRAPHY

Johannes Mach completed a master course in Geodesy at the University of Technology in Graz, Austria. After his graduation in 2001 he started working in the field of applied satellite navigation. It was during a young graduate traineeship at ESA's Galileo Project Office that he took interest in GNSS system design and performance aspects. Since 2004 he is systems engineer at IfEN GmbH, where he is concerned with clock synchronization algorithms for the Galileo integrity system and other integrity related topics.

Ingrid Deuster holds a diploma in Geodesy from the University of Stuttgart. In 1998/1999 she worked as a research associate at the Institute of Photogrammetry and Cartography of the University FAF Munich. She joined IfEN in 1999 as a systems engineer and has worked since then in the field of satellite navigation integrity. Lately she has been dealing with integrity performance experimentation in the GSTB-V1 project and the development of the Experimental IPF for Galileo.

Dr. Robert Wolf holds a degree in Aerospace Engineering from the Technical University of Munich and a Ph.D. from University FAF Munich. He joined IfEN GmbH in 1999. Since 2001 he is company responsible for all integrity related projects. He is also heavily involved in the development of the Processing Facility of the German Galileo Test Bed (GATE).

Theodor Zink received a diploma in Electrical Engineering from the University of Erlangen-Nuremberg in 1995 before becoming research associate at University FAF Munich. He joined IfEN GmbH in 2001, where he has been working on receiver autonomous integrity monitoring (RAIM) and other GNSS related research and development activities.

ABSTRACT

Precise synchronization of monitoring station clocks is a crucial part of a real-time integrity monitoring system as foreseen for Galileo. Stringent requirements on accuracy and robustness in the presence of narrow performance margins raise the need to identify an optimal algorithm for the efficient computation of synchronization parameters.

In this context several types of candidate clock filter algorithms have been studied. Two different approaches can be distinguished:

The first one attempts to gain optimality by processing all available range measurements in an integrated Kalman Filter containing all involved parameters together. It has been implemented and tested in a so-called "Global Filter" version (including estimation of SISE parameters) and a "Common View" version (avoiding SISE parameter estimation by use of range differences).

The alternative approach uses simple least-squares adjustment to estimate the instantaneous clock offsets of all stations. In a second step the clock offsets can be smoothed separately for each station, either by means of Kalman filtering or a fixed gain linear filter.

The paper describes the main features of the filter algorithms and tries to assess their performance based on results of Monte-Carlo simulations. Special focus is put on the discussion, why an integrated Kalman Filter does not seem to be the ideal choice, and on the advantages of the simpler alternative approach.

INTRODUCTION

A GNSS Integrity Monitoring System basically consists of a network of monitoring stations and a data processing center, where the range and phase measurements collected by the stations are used to produce the desired integrity information (alert flags plus monitoring accuracy measures).

In case of Galileo, the so-called Integrity Processing Facility (IPF) will once per second process data from about 40 globally distributed monitoring stations. In each processing cycle the following main computation steps have to be performed:

- Pre-processing of raw measurements
- Synchronization
- SISE estimation & integrity check

This article concentrates on the synchronization step. It has the task to correct the pre-processed pseudo-ranges for the station clock biases. This means that the clock offsets themselves are only intermediate results. However, their accuracy has a direct impact on the accuracy of the post-clock ranges and on the resulting estimation accuracy of the Signal-in-Space Error (SISE). In fact the synchronization can be regarded as the first part of a two-step estimation process!

One of the key performance parameters of the integrity monitoring is the Signal-in-Space Monitoring Accuracy (SISMA), which is defined as the projection of the vector SISE estimation error on the worst user location in the satellite footprint. Of course, the SISMA performance is mainly driven by the accuracy of the monitoring measurements, but it also depends on the used algorithms.

The Galileo requirements foresee nominal SISMA values of 0.70 m or better. Simulations based on current range error specifications have shown that this is a very challenging goal. Therefore, given the situation that the SISE estimation step itself seems to be straightforward (instantaneous least squares estimation) there is a strong motivation to find an optimal synchronization algorithm.

The following criteria can be established for the selection of an appropriate algorithm:

- Accuracy (0.7 ns or better)
- Robustness (to feared events such as clock jumps)
- Short convergence time
- Provision of valid accuracy measures
- Computation time (tenths of a second)

It must be kept in mind that the IPF synchronization is a real-time function. This is a major difference to the kind of synchronization that is performed in the framework of the orbit determination.

CLOCK FILTERING BASICS

The basic idea of clock filtering is to derive benefit from the stability of the atomic clocks that are going to be used in the monitoring stations. Due to this stability it is possible to predict the station clock offsets at the current epoch based on the clock offsets of previous epochs. The predicted clock offsets are valuable information that can be used to support the estimation by increasing the redundancy of the corresponding equation system. Furthermore, the range measurements are subject to noise and filtering can be used to reduce the effect of this noise on the synchronization.

The best method from an integrity point of view would be purely instantaneous estimation of all unknowns (clocks and SISE). There would be no need to consider filtering, if the monitoring measurements were sufficiently accurate to reach the SISMA requirement with instantaneous estimation! This, however, is currently not the case.

Two different concepts of clock filtering have been considered in the study:

- Integrated Kalman filtering

All unknown parameters of the entire network are included in the state vector of one large Kalman filter.

- Least-squares estimation with station-wise filtering

Instantaneously estimated clock offsets are fed into separate, small clock filters.

Both approaches and their variations are based on the following, simple linear models for clock and satellite parameters.

Two-Parameter Clock Model

For our purpose, the behavior of an atomic clock can be modeled with two state vector elements: clock offset (phase) and clock drift (frequency offset).

$$x^{clk} = \begin{bmatrix} x_{offset} \\ x_{drift} \end{bmatrix}$$

The dynamic model is defined by the transition matrix F that relates the states of two epochs k-1 and k:

$$x_k = \Phi^{clk} x_{k-1}$$

$$\Phi^{clk} = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} \qquad \Delta t = t_k - t_{k-1}$$

The stochastic clock model is defined by the process noise matrix Q:

$$Q^{clk} = \begin{bmatrix} q_1 \Delta t + \frac{1}{3} q_2 \Delta t^3 & \frac{1}{2} q_2 \Delta t^2 \\ \frac{1}{2} q_2 \Delta t^2 & q_2 \Delta t \end{bmatrix}$$

The coefficients q1 and q2 are the diffusion constants of the two random processes “White Noise Frequency Modulation” (WFM) and “Random Walk Frequency Modulation” (RWFM), respectively. Chaffee (1987) established a simple relationship for the Allan variance as function of q1, q2, and the integration time τ:

$$\sigma_y^2(\tau) = \frac{q_1}{\tau} + \frac{q_2 \tau}{3}$$

Based on this formula the diffusion constants are most conveniently determined from a logarithmic diagram of the clock's Allan variance or its square root, the Allan deviation. In a logarithmic Allan deviation diagram the WFM and RWFM components are mapped to straight lines of inclination –1/2 and +1/2, as shown in the figure below. Note that the Flicker Frequency Modulation (FFM) cannot be exactly modeled by the considered model. It can only be taken into account by appropriate inflation of the two other coefficients.

The described model is common practice in the precise timing community, cf. Hahn (1999).

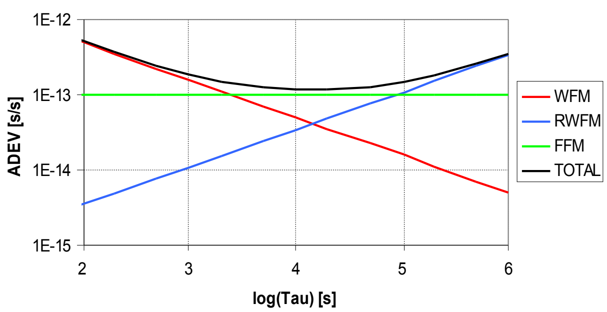

Fig. 1: Example Allan deviation (ADEV) diagram of an atomic frequency standard showing White Noise (WFM), Random Walk (RWFM) and Flicker Frequency Modulation (FFM)

SISE Parameter Model

For the satellite errors a 3-parameter model has proven useful, where the satellite clock and radial orbit error is combined to one variable. This is due to the fact that these two error components cannot be separated with reasonable accuracy, unless long orbit arcs are processed as in the orbit determination. Moreover they have almost identical effects on the user range errors.

$$x^{sat} = \begin{bmatrix} x_{rad(+clk)} \\ x_{atr} \\ x_{ctr} \end{bmatrix} \begin{matrix} \text{... Radial \& clock error} \\ \text{... Along-track error} \\ \text{... Across-track error} \end{matrix}$$

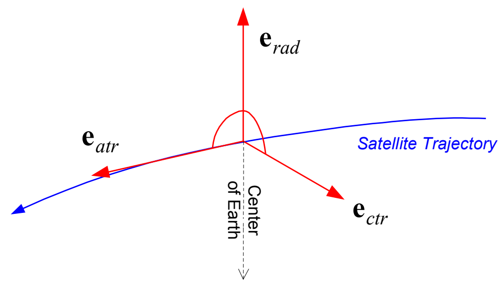

Fig. 2: Orthogonal unit vectors of radial (rad), along-track (atr) and across-track (ctr) SISE components

In order to account for the slow variation of orbit errors, each of the three error components is modeled as a first order Gauss-Markov random process:

$$\Phi^{Sat} = \begin{bmatrix} k_{rad} & 0 & 0 \\ 0 & k_{atr} & 0 \\ 0 & 0 & k_{ctr} \end{bmatrix}$$

$$\mathbf{Q}_i^{Sat} = \begin{bmatrix} \sigma_{rad}^2\left(1-k_{rad}^2\right) & 0 & 0 \\ 0 & \sigma_{atr}^2\left(1-k_{atr}^2\right) & 0 \\ 0 & 0 & \sigma_{ctr}^2\left(1-k_{ctr}^2\right) \end{bmatrix}$$

with

$$k_i = \begin{cases} \tau_{corr,i} > 0: & \exp\left(-\Delta t / \tau_{corr,i}\right) \\ \tau_{corr,i} = 0: & 0 \end{cases}$$

A first order Gauss-Markov process is uniquely defined by its variance and its correlation time (τ_{corr}). For zero correlation times it is equivalent to Gaussian white noise.

THE INTEGRATED KALMAN FILTER

The state vector of the integrated Kalman filter contains the parameters of all stations and satellites altogether. Each station clock and each satellite is represented by a block of variables as introduced above.

$$x^T = \begin{bmatrix} x_1^{clk} & \cdots & x_{n1}^{clk} & x_1^{sat} & \cdots & x_{n2}^{sat} \end{bmatrix}$$

Accordingly the corresponding transition matrix and process noise matrix are formed as block-diagonal matrices with the diagonal blocks as defined for each individual clock and satellite.

$$\Phi = \begin{bmatrix} \Phi_1^{clk} & \cdots & 0 & & & \\ \vdots & \ddots & 0 & & 0 & \\ 0 & 0 & \Phi_{n1}^{clk} & & & \\ & & & \Phi_1^{sat} & \cdots & 0 \\ & 0 & & \vdots & \ddots & 0 \\ & & & 0 & 0 & \Phi_{n2}^{sat} \end{bmatrix}$$

$$Q = \begin{bmatrix} Q_1^{clk} & \cdots & 0 & & & \\ \vdots & \ddots & 0 & & 0 & \\ 0 & 0 & Q_{n1}^{clk} & & & \\ & & & Q_1^{sat} & \cdots & 0 \\ & 0 & & \vdots & \ddots & 0 \\ & & & 0 & 0 & Q_{n2}^{sat} \end{bmatrix}$$

Using the complete parameter model, the pre-clock range residuals can be directly processed as observations. By a pre-clock range residual we understand the difference of the pre-processed pseudo-range minus the range that is computed from the known station coordinates and the broadcast satellite navigation message.

$$\Delta\rho_j^i = \rho_j^i - R_j^i$$

The corresponding observation equation has a coefficient vector, in which only the blocks of the involved station (i) and satellite (j) have non-zero elements:

$$h_{i,j} = \begin{bmatrix} \cdots & h_i^{clk} & \cdots & \cdots & h_j^{sat} & \cdots \end{bmatrix}$$

with

$$h_i^{clk} = \begin{bmatrix} 1.0 & \Delta t \end{bmatrix} \qquad \Delta t = t_{Rx} - t_k$$

$$h_j^{sat} = \begin{bmatrix} 1.0 & \left(e_j^{i^T} e_{atr}^i \right) & \left(e_j^{i^T} e_{ctr}^i \right) \end{bmatrix}$$

The coefficients for the along-track and across-track SISE parameters are computed as scalar products of the line-of-sight unit vector and the respective unit vectors in across-track and along-track direction. The coefficient for the radial & clock component is ambiguous, but it appears justified to set it to the derivative of the clock error (= 1) as the clock prediction error is usually larger than the radial orbit error and the difference is anyway small.

Based on this observation model, the well-known Kalman recursion algorithm can be used to update the state vector and state vector covariance matrix at each update epoch (cf. Brown, 1997).

(1) Compute Kalman gain matrix (K):

$$K_k = P_k^- H_k^T (H_k P_k^- H_k^T + R_k)^{-1}$$

(2) Update estimate (x) with current measurements (z):

$$\hat{x}_k = \hat{x}_k^- + K_k (z_k - H_k \hat{x}_k^-)$$

(3) Update state vector covariance matrix (P):

$$P_k = (I - K_k H_k) P_k^-$$

(4) Project ahead to epoch k+1:

$$\hat{x}_{k+1}^- = \Phi_k \hat{x}_k$$

$$P_{k+1}^- = \Phi_k P_k \Phi_k^T + Q_k$$

(Predicted quantities are marked by a superscript minus.)

To start the recursion an initial state has to be defined. The elements of the state vector can be simply set to zero. The initial covariance matrix (P_0) is a diagonal matrix with appropriate variances as diagonal elements.

In the above Kalman equations the observation matrix (H), the observation vector (z) and the measurement covariance matrix (R) represent a set of (one or several) observations. However, in our observation model all systematic errors are modeled as parameters. The remaining Rx measurement error is purely line-of-sight dependent. Consequently the observations can be regarded as uncorrelated and it is possible to process them separately, one by one. This method avoids the matrix inversion and hence allows for significantly shorter computation times. In fact there are various alternative possibilities to mechanize the Kalman filter equations. More details can be found in Bierman (1977).

"Global Filter" vs." Common View"

The integrated Kalman filter can be varied with respect to the observation model. The algorithm presented in the preceding section is designed to estimate both station and satellite parameters in one single process. It is therefore sometimes referred to as "Global Filter" (GF).

Slightly different from that, the "Common View" (CV) technique, which is used for time transfer in precise timing, uses the difference of range residuals of two stations (i, j) to a common satellite (k) as observations.

$$\Delta d_{i,j}^k = \left(\rho_i^k - R_i^k \right) - \left(\rho_j^k - R_j^k \right)$$

The CV technique allows to significantly reduce the size of the state vector, because the satellite related parameters are no longer necessary. The radial & clock SISE parameter is fully eliminated by the differencing of the linear equations. The tangential orbit errors can be moved from the parameter side to the error side of the equation system.

It is interesting to observe that the two observation models yield identical results when used for instantaneous estimation (under the condition that the radial SISE parameter is estimated freely by the GF, i.e. its *a priori* variance is set to a large value). Nevertheless there is one significant difference in case of filtering: The CV does not allow specifying an auto-correlation model for the orbit errors. This causes a considerable model error, since orbit errors have strong auto-correlations.

A surprising result is obtained from the comparison of the execution times of the two algorithms. Although the CV algorithm has a much smaller state vector (for the same configuration), it does not economize computation time. This is due to the fact that the CV range differences are no longer uncorrelated and therefore one-by-one processing of scalar observations is not permitted in this case.

LEAST-SQUARES ESTIMATION WITH STATION-WISE FILTERING

A promising alternative to the integrated Kalman filter is the combination of instantaneous least-squares estimation with subsequent station-wise filtering.

For the instantaneous least-squares adjustment the same observation models can be used as for the integrated Kalman filter. The only difference is that it does not make sense to include clock drift parameters, as a clock drift cannot be estimated from data of only one epoch.

The inversion of the normal equations is just one possibility to perform the adjustment:

$$P_k = (H_k R_k^{-1} H_k^T)^{-1}$$

$$x_k^{inst} = P_k H_k^T R_k^{-1} z_k$$

In practice algorithms based on Cholesky or QR decomposition are more efficient and have better numerical stability.

From the results of this first step the instantaneous clock offsets and their estimation variances are obtained. In the second step these instantaneous offsets are smoothed by means of separate smoothing filters. In the following two propositions are made for this kind of smoothing.

One-clock Kalman Filter with noise parameter

The two-parameter linear clock model is augmented by a noise parameter that allows modeling of colored noise. The colored noise parameter is useful, because the pre-processed ranges are affected by colored noise errors and these errors propagate to the estimated clock offsets.

$$x^{aug} = \begin{bmatrix} x_{offset} \\ x_{drift} \\ \varepsilon_{noise} \end{bmatrix}$$

The simplest way to model colored noise is again a first order Gauss-Markov model defined by variance and correlation time. This leads to the following transition and process noise matrices:

$$\Phi^{aug} = \begin{bmatrix} 1 & \Delta t & 0 \\ 0 & 1 & 0 \\ 0 & 0 & k_\varepsilon \end{bmatrix} \quad \Delta t = t_k - t_{k-1}$$

$$Q^{aug} = \begin{bmatrix} q_1 \Delta t + \frac{1}{3} q_2 \Delta t^3 & \frac{1}{2} q_2 \Delta t^2 & 0 \\ \frac{1}{2} q_2 \Delta t^2 & q_2 \Delta t & 0 \\ 0 & 0 & \sigma_\varepsilon^2 \left(1 - k_\varepsilon^2\right) \end{bmatrix}$$

$$k_\varepsilon = \begin{cases} \tau_{corr,\varepsilon} > 0: & \exp\left(-\Delta t / \tau_{corr,\varepsilon}\right) \\ \tau_{corr,\varepsilon} = 0: & 0 \end{cases}$$

Now the instantaneous clock offsets can be processed using the usual Kalman recursion with the following observation equation:

$$h_k x_k^{aug} = z_k = x_k^{inst}$$

$$h_k = \begin{bmatrix} 1.0 & \Delta t & 1.0 \end{bmatrix} \quad \Delta t = t_{Rx} - t_k$$

The variance of this “pseudo-observation” reflects only additional white noise. If there is no white noise, the observation variance is zero.

One-clock Fixed Gain Smoothing Filter

The Kalman Filter automatically computes optimal gains. It can be tuned only via the configuration of the process noise, which can be a difficult task.

Much easier handling is offered by a smoothing filter with fixed gain factors for drift and offset. It uses the following update formulas for offset and drift:

$$x_k^{off} = \left(1-\overline{k}_k^{off}\right)\left(x_{k-1}^{off} + \Delta t \cdot x_{k-1}^{drf}\right) + \overline{k}_k^{off}\, x_k^{inst}$$
$$x_k^{drf} = \left(1-\overline{k}_k^{drf}\right) x_{k-1}^{drf} + \overline{k}_k^{drf}\left(x_k^{inst} - x_{k-1}^{inst}\right) / \Delta t$$
$$\Delta t = t_k - t_{k-1}$$

For both clock offset and drift the effective gain factors are computed themselves recursively. This recursion via a weight variable (w) has the purpose to produce gain factors that as smoothly as possible approach the asymptotic gain factors (k).

$$w_k^i = \left(1-k^i\right) w_{k-1}^i + k^i$$
$$\overline{k}_k^i = k^i / w_k^i$$

The asymptotic gain factors are conveniently configured by means of filter time constants (T) for both offset and drift smoothing:

$$k^i = 1 - \exp\left(\Delta t / T_i\right) \approx 1 / T_i$$

The characteristic feature of this filter is that the drift update does not depend on a predicted offset. In this way disturbing resonance effects between the drift and offset smoothing are avoided.

PERFORMANCE ASSESSMENT

Monte-Carlo simulations were used in order to assess the performance of the considered synchronization algorithms. In these simulations all relevant errors, i.e. clock noise, orbit errors and measurement errors were generated by means of appropriate random processes based on random number generation.

Comparison of the estimated clock offsets with the simulated "true" clock offsets results in "true" synchronization errors that can be statistically analyzed.

The following basic simulation assumptions roughly reflect the current baseline for Galileo:

- Constellation: Walker 27/3/1, 10 days repeat cycle
- Station network: 40 stations world-wide
- Range error: 0.30 m (90 deg) to 0.80 m (10 deg) for pre-processed ranges (Galileo L1/E5b ionospheric-free combination)
- Clock noise (Rb clock):
  - Frequency White Noise: 1.5e-11 (ADEV 1 s)
  - Frequency Random Walk 2.0e-15 (ADEV 1 s)
- Time Reference: Master clock (Toulouse)
- Orbit errors: Gauss-Markov (correlation time 3600 s)
- Simulation duration: 12 hrs.

The scenarios were set up to cover the given algorithm alternatives and are summarized in Tab. 1. Note that the state parameters associated with the tangential SISE components (along-track, across-track) were constrained to zero with 1 m uncertainty for all scenarios except the "free estimation" case. For the radial & clock SISE component the constraint was minimized by setting the corresponding standard deviation to a large value. This was done to avoid a constraint towards the reference time of the navigation message, which does not necessarily have to coincide with the IPF time reference. This is not relevant for the Common View algorithm, for which the radial SISE error plays a minor role.

Tab. 1: Description of basic simulation scenarios

Scenario	Description	Upd. Int. (s)
Inst_Free	Instantaneous estimation with quasi-free orbit estimation; $\sigma_{rad} = \sigma_{atr} = \sigma_{ctr} = 100$ m	1
Inst_Tan_1m	Instantaneous estimation with soft constraint for tangential orbit errors; $\sigma_{rad} = 100$ m; $\sigma_{atr} = \sigma_{ctr} = 1$m	1
GF*	Integrated Kalman filter using "Global Filter" method	1 10 60
CV*	Integrated Kalman filter using "Common View" method	1 10 60
LS & KF*	Instantaneous least-squares estimation with station-wise Kalman filter	1
LS & Smo*	Instantaneous least-squares estimation with station-wise fixed gain smoothing filter; Filter constant for offset = 5 s; Filter constant for drift = 500 s	1

*) Using same *a priori* orbit errors as "Inst_Tan_1m"

The White Noise Measurement Error Case

In a first step, let us assume the range measurement errors to be white, i.e. uncorrelated with respect to time. This allows us to verify the function of the algorithms under "ideal" conditions. Later it will be shown that the performance in presence of colored noise can considerably differ from the purely theoretical white noise case.

An overview of the statistical results of the white noise scenarios is given by Fig. 3. For each scenario the a priori standard deviation as computed by the algorithm is depicted (red bar) as well as the RMS value of the actual "true" synchronization errors (blue bar). Since the a priori standard deviations are used as input for the SISMA computation, they should be representative for the true errors (or conservative).

The first two scenarios in the bar chart refer to the instantaneous estimation without any filtering. Obviously the performance that can be reached without using any a priori information about the orbit error (free estimation) is rather poor (1.8 ns). This error can be significantly improved (to 1.0 ns) by imposing a soft constraint on the tangential orbit errors. However, this also means that the IPF synchronization algorithm depends on the quality of the navigation message. In spite of this concern the performance of the "Inst_Tan_1m" scenario will be used as our benchmark for the assessment of the other algorithms, since the free instantaneous estimation, which would be ideal from an integrity point of view, is clearly not accurate enough.

The results of the integrated Kalman filter scenarios are shown in Fig. 3 as well. For both Global Filter (GF) and Common View (CV) several update intervals have been considered. In both cases one can see that, in the white noise case, the synchronization error becomes considerably smaller with decreasing update interval. Besides, the GF generates perfectly consistent standard deviation values under these circumstances. This is in agreement with our expectations as for the GF the used estimation models are identical to the truth reference models. Unlike that, the CV method produces too optimistic standard deviations and at the same time larger true errors, which is a consequence of the model error regarding the SISE auto-correlation as pointed out before.

Above all, the results seem to indicate that for white measurement noise the use of an integrated Kalman filter would be very effective, and that the Global Filter potentially outperforms the Common View thanks to the better SISE modeling.

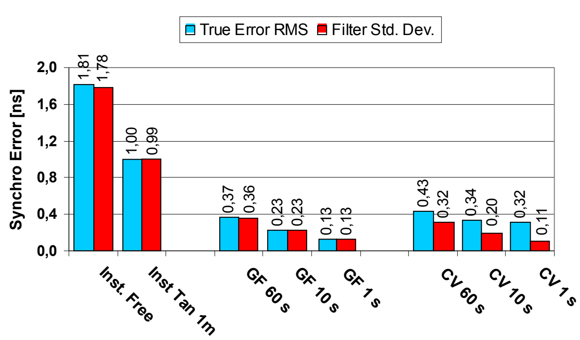

Fig. 3: Statistics of synchronization performance for white observation noise

As a matter of fact, every clock filter needs some time until it reaches optimal performance ("steady state"). As shown by the exemplary time series in Fig. 4 the convergence time is roughly 1-2 hours. Although the estimation of the clock offset is fairly good (1 ns) right from the beginning, it takes a certain time span to obtain a good estimation of the clock drift.

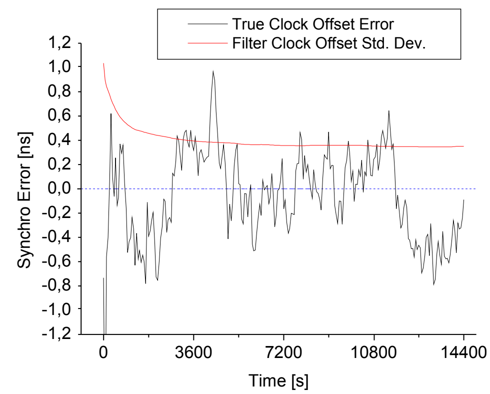

Fig. 4: Time series of true synchronization error and consistent standard deviation from filter (GF_60s with white observation noise, station Kourou)

The Colored Noise Measurement Error Case

For a meaningful assessment it is necessary to test the algorithms under more realistic conditions. This particularly means that the auto-correlation of range observation errors has to be taken into account. The auto-correlation of measurements has two main reasons: Firstly, the raw code range measurements themselves have a strong colored noise component due to multipath effects. Secondly the raw pseudo-ranges are smoothed within the pre-processing using the phase measurements (Hatch filter).

Therefore all simulations were repeated under a colored measurement noise scenario. As a simple approximation, the measurement error on each line-of-sight was simulated as a first order Gauss-Markov process with a correlation time of 600 s. The interesting outcome of these simulations is presented in Fig. 5.

As expected, the two instantaneous estimation scenarios are not affected by the time correlation and therefore yield the same results as for white noise (apart from small statistical deviations). Their a priori standard deviations are still consistent with the true errors.

For the integrated Kalman we have a completely different picture: There is a strong mismatch between the predicted accuracy and the true error statistics. The severity of this discrepancy is a function of the chosen filter update interval, aggravating for short update intervals. Furthermore, even the absolute synchronization performance deteriorates for shorter update intervals, in spite of the fact that the accumulation of clock noise should be unfavorable for long update intervals.

A secondary but nonetheless surprising aspect is that the performance degradation in presence of colored noise is more important for the GF than for the CV. Both perform badly, but in contrast to the white noise here the CV is slightly better compared to the GF.

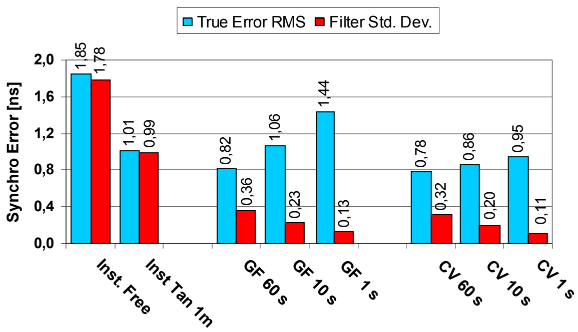

Fig. 5: Statistics of synchronization performance for colored observation noise (600 s correlation time)

We conclude that the model errors with regard to the observation error auto-correlation have such a strong impact that the integrated Kalman filter performs hardly better than the instantaneous estimation and beyond that does not provide reasonable filter standard deviations.

This raises the question, whether the second approach, combining instantaneous LS estimation with subsequent station-wise filtering, is able to improve the performance. The statistics of the corresponding simulations, both for the white noise (WN) and colored noise (CN) case, are compiled in Fig. 6.

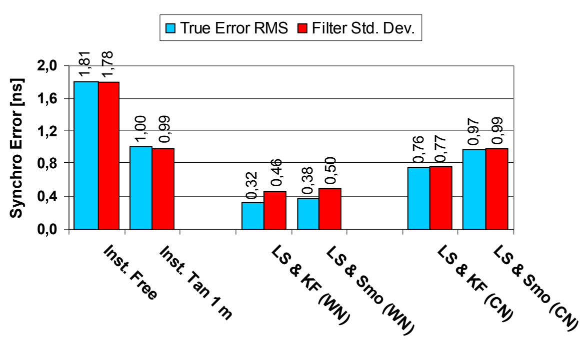

Fig. 6: Statistics of synchronization performance for LS estimation with station-wise filtering; center: white noise scenario; right: colored noise scenario

Again, performances are very appealing for white noise measurement errors (0.32 ns), although comparison with the Global Filter (0.13 ns) clearly reveals the sub-optimality of the two-step approach. Anyway we are more interested in the colored noise results: Here, the method of separate Kalman filtering for each station indeed seems to result in a considerable improvement with respect to the unfiltered instantaneous estimation (0.77 ns versus 1.00 ns). This is the best result among all algorithms! Furthermore, the use of an additional noise parameter (with consistent auto-correlation model) leads to highly representative a priori standard deviations. Contrary to that, the benefit of a simple first-order smoothing filter turns out to be almost negligible (0.99 ns).

DETAILED ANALYSIS

The Colored Noise Problem

The obvious question is: What is the reason for the poor performance of the integrated Kalman filter in the presence of colored measurement noise?

A closer look at the statistics in Fig. 5 gives us a good hint: The true error RMS values and the mean filter standard deviations seem to approach each other for larger update intervals. Indeed further experimentation has shown that the mismatch largely disappears, when the update interval is equal or greater the double measurement correlation time. This is not surprising, since in that case the processed measurements are almost uncorrelated.

It can be concluded that the discrepancy roughly depends on the ratio of update interval and measurement correlation time. The smaller the update interval, the more strongly consecutive measurements are mutually correlated. The Kalman filter, however, implicitly expects the observation errors of different epochs to be uncorrelated. It assumes equal information content of each measurement of a given line-of-sight, which does not reflect the true situation. For this reason the state vector covariance matrix becomes much too optimistic and is not consistent with the actual estimation accuracy.

Now the gain factors that are computed by the Kalman algorithm at each observation update are a function of the observation variances and the current state vector covariance matrix. Too small filter variances result in smaller gain factors, i.e. lower weights for new measurements, than actually needed. Consequently the computed Kalman gains are no longer optimal. The filter becomes too rigid and follows the actual clock signal only with a considerable lag. As seen in the simulations, this effect can even lead to worse performances than those of the instantaneous estimation.

A similar explanation can be given for the particularly bad performance of the Global Filter as opposed to the Common View. From analysis of the estimated SISE parameters one finds that the Global Filter actually "misinterpretes" the slowly varying range errors as orbit errors. The resulting very poor SISE estimates do more bad than good.

Looking for a Workaround

Several possible counter-measures have been considered in order to resolve or mitigate the colored noise problem:

- Option 1: Increase update interval

According to Fig. 5 increasing the filter update interval somewhat improves the situation. However, a filter update interval of the order of the measurement correlation time (e.g. 600 s) is not acceptable. Moreover this method is only useful, as long as all clocks really behave within their specification. In case of higher clock dynamics again a shorter update interval would be preferable.

- Option 2: Inflate observation variances

Another idea is to account for the reduced content of information per measurement by inflating the observation variances by a certain factor. In fact this methods allows to obtain higher filter variances. On the other hand it does not improve the estimation errors, because the inflated measurement variances result in even lower gain factors and hence make the filter even more rigid.

- Option 3: Augment state vector by noise parameters

In principle it is also possible to model the colored noise errors by appropriately augmenting the state vector with noise parameters. Unfortunately this would require one additional parameter per line-of-sight. The resulting state vector would have up to 400 noise parameters. Therefore, having in mind the strict computation time constraints, this is not a feasible approach either.

Summing up, none of these modifications of the integrated Kalman filter concept actually appears appropriate.

A Simple but Promising Alternative

On the other hand, the proposed alternative approach to combine instantaneous least-squares estimation with separate station-wise clock filtering gives fairly good performance results. It is actually a simplified version of the above-mentioned option of state vector augmentation, but tries to model the effect of the colored noise on the estimated clock offsets rather than the line-of-sight errors themselves. This is a reasonable approximation, since the least-squares adjustment is a linear estimator. If all measurement errors can be characterized as Gauss-Markov processes with a common, constant correlation time, then the estimation error will also be Gauss-Markov with the same correlation time. As indicated by the simulation results, the filter standard deviations produced by this algorithm are representative.

Another advantage of this two-step approach is the reduction of complexity that enables fast computation but also offers better robustness. Whereas in case of the integrated Kalman filter a local feared event such as a station clock jump has a global impact on the state vector and in the worst case can corrupt the entire filter, in the two-step algorithm only the separate filter of the clock that has undergone the jump will be affected.

CONCLUSIONS

As a major result, the presented study has shown that an integrated Kalman filter (including all stations and possibly satellites) under realistic circumstances does not yield the expected significant performance improvement. Besides it does not provide representative filter variances that are required for the SISMA computation. This phenomenon is primarily caused by an unresolved modeling problem regarding the strong auto-correlation of measurement errors.

As an alternative, a simple two-step algorithm combining instantaneous least-squares estimation and subsequent, station-wise clock filtering has been introduced. Out of two versions, in particular the one employing Kalman filtering with one additional noise parameter per station has proven to be promising. The reported simulations indicate a potential improvement of about 25% for this method with respect to the unfiltered instantaneous estimation. It is characterized by reduced complexity and vulnerability, but nevertheless offers consistent filter variances.

In general it can be stated that the colored noise affecting pre-processed pseudo-ranges imposes limitations on the possible benefit of clock filtering in the context of GNSS integrity monitoring. Further research and testing with real data will be needed to find the best solution.

ACKNOWLEDGEMENTS

A part of the software that has been used to produce the results presented in this paper has been developed by IfEN in the framework of the ESA projects GSTB-V1 and Galileo C0 under subcontracts to Alcatel Space.

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