Making GNSS Integrity Simple and Efficient – A New Concept Based on Signal-in-Space Error Bounds

Johannes Mach, Ingrid Deuster, Robert Wolf, Wolfgang Werner
IfEN GmbH, Germany

BIOGRAPHY

Johannes Mach completed a master course in Geodesy at the University of Technology in Graz, Austria. After his graduation in 2001 he started working in the field of applied satellite navigation. It was during a young graduate traineeship at ESA's Galileo Project Office that he took a special interest in GNSS system design and performance aspects. Since 2004 he has been working as systems engineer in the integrity systems department of IfEN GmbH, where he is concerned with algorithm design, software development and performance analysis for several integrity related projects.

Ingrid Deuster holds a diploma in Geodesy from the University of Stuttgart. She joined IfEN in 1999 as a systems engineer and has been working since then in the field of GNSS integrity processing. Lately she has been dealing with integrity performance experimentation in the GSTB-V1 project and the development of IfEN's Experimental IPF for Galileo.

Robert Wolf holds a degree in Aerospace Engineering from the Technical University of Munich and a Ph. D. from University FAF Munich. He joined IfEN GmbH in 1999. Since 2001 he is company responsible for all integrity related projects. Among other topics he is currently involved in the development of the Processing Facility of the German Galileo Test Bed (GATE).

Wolfgang Werner received a diploma in Computer Science from the University of Technology in Munich in 1994 and a Ph. D. from the University FAF Munich in 2000. Since 1999 he is Technical Director of IfEN GmbH. He led the algorithm development for the EGNOS Check Set and also worked on integrity algorithms in the framework of several Galileo projects. Based on this experience he tracks and supports all activities in the field of integrity.

ABSTRACT

This publication has the objective to contribute a new concept to the ongoing scientific discussion, which can be regarded as a modified version of the Galileo integrity concept.

The characteristic feature of the concept is the dissemination of instantaneous Signal-in-Space Error Bounds (SISE-B) instead of the Signal-in-Space Monitoring Accuracy (SISMA). This small but effective modification results in a significant reduction of performance losses that are inherent to the Galileo concept. Furthermore a modified definition of SISA overbounding is introduced, which is more favorable to verification. The optimized combination of statistical (SISA) and instantaneous (SISE-B) information allows relaxing overbounding requirements and still minimizing the integrity risk.

The paper explains the background and gives high-level specifications of both system and user algorithms. Special focus is put on the user protection approach including integrity and continuity aspects. The expected availability performance is assessed by means of simulation results.

INTRODUCTION

With upcoming new systems such as Galileo there is a strong need to establish a consolidated concept that can serve as a standard for future GNSS integrity services. Such a concept has to meet several important requirements. First of all, its safety has to be strictly justified in terms of integrity and continuity risk, in order to allow GNSS positioning for safety-critical applications. Secondly, it needs to be as efficient as possible thereby offering a maximum of availability for different user groups. Finally, as soon as public or private liabilities are granted by a service, it becomes necessary to allow external verification of its reliability and monitoring of its performance.

What exactly has to be understood by a GNSS integrity concept? Would it not be sufficient to simply specify a guaranteed maximum error for the user position, maybe complemented by warning messages for anomalous situations? Unfortunately not. One of the characteristics of satellite navigation is that user positioning accuracy always depends on several factors such as the system performance, the receiver's ranging accuracy and the individual observation geometry. Consequently a meaningful confidence interval for the position error can only be computed by the user itself, based on some data provided by the system. Therefore a GNSS integrity concept always has to address the entire cycle from the system's integrity monitoring via the definition of the broadcast integrity data down to the user algorithm.

Starting point of the present study is the integrity concept that has been developed for the Galileo safety-of-life integrity service. It has been described in several recent publications (Oehler, 2004; Oehler, 2005), and knowledge of these references will be also helpful for the reader's understanding of this paper.

One of the innovative features of the Galileo integrity concept is the so-called Galileo adaptive user algorithm (Oehler, 2004b), which allows the user to compute his current integrity risk based on the broadcast parameters SISA (Signal-in-Space Accuracy) and SISMA (Signal-in-Space Monitoring Accuracy). Unlike the SBAS/WAAS user algorithm it covers not only the fault-free case but also the single failure case. In this way it also accounts for the additional risk arising from possible biases that are large enough to have an impact yet too small to be detected with sufficient reliability.

However, there are also a few critical points of the Galileo integrity concept. The Galileo user algorithm strongly relies on the assumption that the Signal-in-Space Error (SISE) is statistically overbounded by a Gaussian distribution with standard deviation SISA. Likewise, the monitoring error is supposed to be overbounded by SISMA. As it will be explained in the next chapter, the conventional definition of overbounding does not allow satisfactory empirical verification. It must therefore be regarded as hypothesis that cannot be thoroughly proven. Moreover, the performance of Galileo's SISMA is very sensitive to monitoring station outages, which requires costly extra redundancy of the station network. Bad SISMA performance in turn impairs availability.

Detailed analysis of these issues led to the definition of a new concept that allows relaxing the overbounding requirement and replaces the SISMA parameter by a new, more efficient parameter. Before explaining this concept in detail, in the next chapter a modified definition of overbounding will be given.

OVERBOUNDING DEFINITION

Motivation

User integrity algorithms widely employ the assumption of unbiased (zero-mean) Gaussian error distributions. This is a very convenient model, because it allows very easy computation of the distribution of a linear function of stochastic variables. Real SISE distributions have been shown to be Gaussian-like, cf. Herraiz-Monseco (2001), but nevertheless may differ from an exact unbiased Gaussian distribution. In spite of this fact the use of a Gaussian model is still allowed under certain conditions. To justify this, the concept of statistically overbounding probability distributions has proven useful.

Generally speaking, the overbounding concept allows deriving a conservative characterization of the position error from conservative models of the underlying range error distributions. More specifically, it forms the theoretical basis of the following statement: If one finds for each satellite an unbiased Gaussian distribution that actually overbounds the true SISE, then the true position error (which is a linear combination of the involved satellites' orbit and clock errors) again will be overbounded by a Gaussian unbiased function. The corresponding standard deviation can be computed using standard covariance propagation. The mathematical background of this statement and its detailed conditions can be found in DeCleene (2000).

Conventional definition

The overbounding property in a strict sense is defined as follows:
A probability density function $\overline{p}$ "overbounds in a strict sense" a probability density function p, if for any integration interval [-L, L] the integral of $\overline{p}$ is less than or equal to the respective integral of function p.

$$\int_{-L}^{L} \overline{p}(x)dx \le \int_{-L}^{L} p(x)dx \qquad \forall L$$

The symmetric integral can be regarded as centered cumulative distribution function (cdf). Figure 1 gives a simple example for an overbounding distribution.

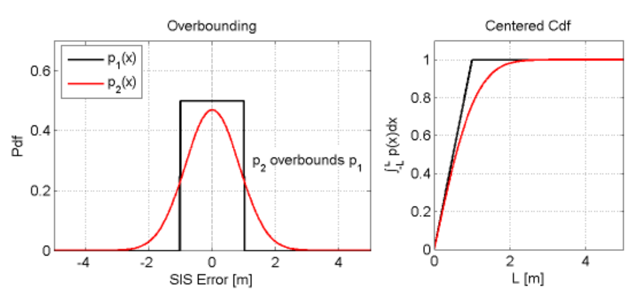

Figure 1: Illustration of the overbounding property

Verification aspects

In the context of a safety-of-life integrity service the overbounding property has to be verified by statistical tests. A straightforward plausibility test would be to compare for various L the number of test samples within the interval [-L, L] with the mean number of samples that is computed from the overbounding model function. Equivalently one can count the samples outside the interval and compare it with the expected number (The necessary test confidence interval is neglected here):

$$n_{Test}(L) \le n_{Model}(L)$$

$$n_{Test}(L) = \text{number of samples x with } |x| > L$$

$$n_{Model}(L) = N\left(1 - \int_{-L}^{L} \overline{p}(x)dx\right)$$

$$N = \text{total number of test samples}$$

However, such a test can never completely verify strict overbounding, as for large L the expected number of samples (n_{Model}) will always get close to zero and therefore no effective test will be possible. In order to achieve reasonable test significance, n_{Model} should be at least in the order of 10. As a consequence the overbounding capability can be verified only up to a certain confidence level. This particularly applies to the verification of SISE overbounding, because orbit and clock errors show strong auto-correlation. Therefore uncorrelated test samples can be collected only with a large sampling interval. Given this fundamental limitation, a less stringent definition of overbounding has to be introduced for the purpose of SISE characterization.

Overbounding with finite confidence level

A probability density function $\overline{p}$ "overbounds with confidence level P" a probability density function p, if for any integration interval [-L, L] with $L \le L_P$ the integral of $\overline{p}$ is less than or equal to the respective integral of function p.

$$\int_{-L}^{L} \overline{p}(x)dx \le \int_{-L}^{L} p(x)dx \qquad \forall L \le L_P \quad \text{such that } \int_{-L_P}^{L_P} \overline{p}(x)dx = P$$

Overbounding with finite confidence level P can be verified with a finite number of independent test samples that is related to P by the following rule of thumb:

$$N \ge 10/(1-P)$$

The modified overbounding definition is an essential prerequisite for the new integrity concept that will be described in the following chapter.

THE SISE BOUND CONCEPT FROM A SYSTEM PERSPECTIVE

Broadcast integrity data

In the proposed concept the user is provided with two integrity-related broadcast parameters per satellite that are defined as follows:

- SISA (Signal-in-Space Accuracy) with confidence level $P_{SISA} = 1 - \alpha$
- SISE Bound (Signal-in-Space Error Bound) with confidence level $P_B = 1 - \beta$

SISA represents the standard deviation of an unbiased Gaussian distribution that statistically overbounds the SIS related pseudorange error for any user in the service area with a specified, constant confidence level P_{SISA}. It is a prediction over the validity period of the current navigation message.

The SISE Bound (B_{SISE}) is a metric bound for the absolute, instantaneous signal-in-space error, which is guaranteed by the system with a specified, constant confidence level P_B. It covers the actual SIS related pseudorange error as observed by any user in the service area throughout a defined continuity period (= critical operation period).

The two parameters provide the user with the following specific information about the actual SIS error:

- $N(0, \sigma_{SISA})$ overbounds SISE with probability P_{SISA}
- $|SISE| \le B_{SISE}$ with probability P_B

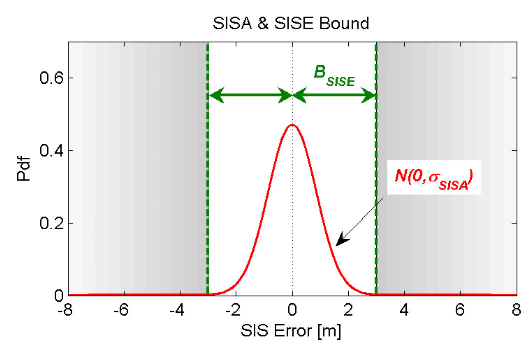

Figure 2: Meaning of SISA (statistical) and SISE bound (instantaneous)

Note the complementary information content of the two broadcast parameters (see also Figure 2):

On the one hand SISA describes the *statistical long-term behavior* of SISE as characterized from a suitable set of historical data. It is therefore disseminated as part of the navigation message with an update interval of typically one to several hours.

On the other hand the SISE Bound represents the *instantaneous* SISE as estimated by the integrity monitoring system taking into account the current monitoring accuracy. It is computed by the real-time integrity processing function and disseminated every second (if needed) as main part of the integrity message.

While SISA basically has the same meaning as in the Galileo concept, the SISE bound actually replaces the broadcast SISMA. In view of the very limited bandwidth for real-time data dissemination, it has to be coded using a look-up table with suitable quantization levels. Considering a bit allocation of 4 bits per satellite, such a look-up table could be defined for example as follows:

Bit sequence	SISE Bound [m]	Bit sequence	SISE Bound [m]
0 0 0 0	Not Monitored	1 0 0 0	6.0
0 0 0 1	2.0	1 0 0 1	7.0
0 0 1 0	2.5	1 0 1 0	8.0
0 0 1 1	3.0	1 0 1 1	9.0
0 1 0 0	3.5	1 1 0 0	10.0
0 1 0 1	4.0	1 1 0 1	12.0
0 1 1 0	4.5	1 1 1 0	15.0
0 1 1 1	5.0	1 1 1 1	Don't Use

Table 1: SISE bound look-up table (example)

For system scalability the numerical values should be multiplied by a common scaling factor that is broadcast with a lower repetition rate.

$$B_{SISE} = k_{Scale} \cdot B_{Table}$$

The "Not Monitored" (NM) flag means that the system momentarily cannot compute a valid SISE bound due to lack of data or a processing failure. The user can use a satellite flagged as NM to finish an already started operation but not to start a new operation.

A "Don't Use" (DU) flag is issued either if the computed bound exceeds the largest entry in the look-up table or if any kind of SIS instability is detected. After reception of a DU flag, the user must immediately exclude the affected satellite from the navigation processing.

SISE bound computation algorithm

In this subsection a simple analytical method is outlined, which ensures that the specified confidence level is achieved. It starts from two typical intermediate results of the integrity processing:

- Estimated scalar SISE (SISE_EST)

This is the immediate output of the instantaneous SISE estimation that is performed by the integrity processing. It is an estimate of the maximum pseudo-range error that can be caused by the ephemeris and clock errors for any user within the satellite footprint. It is computed by projecting the estimated vector SISE (usually three parameters) onto the so-called "worst user location".

- Signal-in-Space Monitoring Accuracy (SISMA)

SISMA is defined as the current standard deviation of the SISE estimation error. It is computed by projecting the covariance matrix of the SISE estimation error (3x3 matrix) onto the worst user location. SISMA is based on *a priori* range error standard deviations that have to be determined for all monitoring stations in a suitable calibration campaign.

Based on these intermediate results, the SISE bound can be computed in a straightforward manner by forming the sum of the absolute value of SISE_EST and a suitable margin, which accounts for the uncertainty of the estimation.

Assuming Gaussian distribution of the (signed) SISE estimation error, the necessary margin is obtained by scaling the SISMA standard deviation to the desired confidence level. An extra margin d_{cont} is added in order to cover SISE throughout the critical operation period.

$$B_{SISE} = \left|SISE_{est}\right| + k_B \cdot \sigma_{SISMA} + d_{cont}$$

A conservative scaling factor k_B can be expressed as symmetric (double-sided) percentile of a normalized Gaussian error distribution.

$$\int_{-k_B}^{+k_B} \frac{1}{\sqrt{2}\sigma} \exp\left(-\frac{x^2}{2}\right) dx = 1 - \beta$$

$$\beta = P\left(\left|SISE_{true}\right| > B\right)$$

All that remains to be done is to map the internally computed bound to one of the look-up table entries, which results in the final broadcast SISE bound.

Note that it is a conservative simplification to first determine separate maximum values for both, SISE and SISMA, before computing the bound. Since in general the two have different worst user locations, it is more precise to evaluate the SISE bound directly from the vector representation of SISE and (matrix) SISMA for a set of grid points in the satellite footprint before searching the overall maximum.

USER PROTECTION

Stochastic model in range domain

The SISA computation must be designed to ensure overbounding of the true SISE distribution. As explained before, the overbounding capability can be verified only up to a certain confidence level P_{verif}. It is not safe to extrapolate the distribution function beyond this confidence level.

Nevertheless it can be stated that any sample with probability P_{verif} belongs to a subset that is strictly overbounded by the model distribution, whereas with a probability of $\alpha_{verif} = 1 - P_{verif}$ the sample belongs to the complementary subset, for which overbounding cannot be guaranteed.

This "overbounding failure" risk arising from the limited verification must be dealt with in the system risk allocation. In fact it has the same impact as the probability of a real SIS failure, which is determined by RAMS analysis. Thus the total risk that the SISA parameter of a certain satellite is not representative can be expressed as sum of overbounding failure and SIS failure probabilities.

$$\alpha = P_{fail,SISA} = \alpha_{verif} + P_{fail,Sat}$$

The resulting stochastic SISE model, which is used by the user algorithm, considers two cases (see also Figure 3):

- With a probability of $1-\alpha$ any considered sample belongs to an ensemble that is strictly overbounded by an unbiased Gaussian distribution with standard deviation SISA.
- With a probability of α the considered sample is not covered by the overbounding distribution, and no statement can be made about its stochastic behavior.

In fact the samples covered by the overbounding are also constrained to the interval between the percentiles that correspond to the SISA confidence level. But this information is not used in the following, because the Gaussian model is much more convenient than a discontinuous probability function.

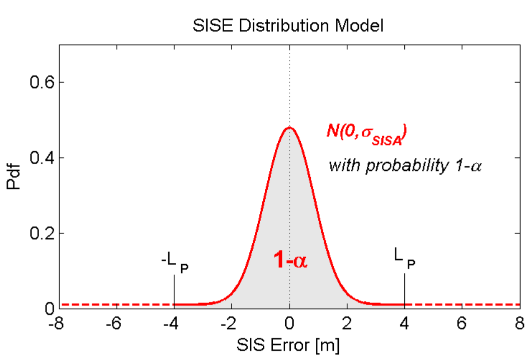

Figure 3: Stochastic SISE model

Integrity risk in position domain

In the previous chapter it was specified that for both SISA and SISE bound there is a residual failure risk that has to be accounted for in the system risk allocation. The goal now is to establish a logic that shows that the overall risk of a so-called HMI event (Hazardous Misleading Information) at user level is below the required value.

In the user algorithm the errors of all used satellites are mapped to the position domain by way of linear combination. According to probability calculus, the resulting position error is again overbounded by a Gaussian distribution with confidence level:

$$P_{Pos} = \left(P_{SISA}\right)^N = (1-\alpha)^N$$

$N =$ number of satellites involved in position solution

The complementary failure risk is equal to the probability of having at least one failure.

$$P_{fail,Pos} = 1-(1-\alpha)^N = P(x \geq 1) = \sum_{i=1}^{N} P(x=i)$$

$x =$ number of failures

It can be split into the probability to have exactly one failure and the probability to have two or more failures.

$$P(x \geq 1) = P(x=1) + P(x \geq 2)$$

For these two probabilities the following conservative estimates can be found:

$$P(x=1) = \binom{N}{1}\alpha(1-\alpha)^{N-1} \leq N \cdot \alpha$$

$$P(x \geq 2) \leq \binom{N}{2}\alpha^2 = \frac{N(N-1)}{2}\alpha^2$$

The risk in the fault-free case has to be controlled by the user algorithm. It can be computed based on the Gaussian model. The user algorithm limits this risk to a maximum value P_{HMI} by issuing a warning (i.e. declaring unavailability of integrity), whenever the computed value exceeds P_{HMI}. Note that this is safe, even if P_{HMI} is smaller than it should be allowed according to the confidence level, because the overbounding failure risk is already accommodated in another term. The resulting total system-related user risk must be below 2e-7/150s, which is the top-level requirement for precision approach in civil aviation as defined in RTCA (2001).

$$P_{HMI,total} = P_{HMI,FF} + P_{fail,Pos} \le \underbrace{P_{HMI,FF}}_{fault-free} + \underbrace{\binom{N}{1}\alpha}_{single\ failure} + \underbrace{\binom{N}{2}\alpha^2}_{multiple\ failure}$$

This equation clearly shows that for a user algorithm that only covers the fault-free case (as for example the SBAS/WAAS algorithm), the failure risk per satellite would have to be demonstrated to be in the area of 1e-8/150s. Otherwise the impact of the linear term would be too large.

Since this is hardly feasible, the user algorithm must additionally cover at least the single-failure case (FM), and that is the moment, where the SISE bound enters. In each of the *N* sub-cases *FM(i)* one can assume that:

- Satellite (*i*) is not covered by the overbounding, but the instantaneous SISE is nevertheless constrained by the broadcast SISE bound with a residual risk β.
- All other satellites are covered by strictly overbounding Gaussian distributions, characterized by the respective SISA values.

Observe that the risk, that another satellite is affected by a failure, is already included in the multiple-failure probability term. Based on this FM model the SISE bound user algorithm covers all single-failure cases except those, where the SISE bound fails too, i.e. the true error is greater than the bound. In this way the impact of the one-failure case is reduced by factor β, at least under the condition that α- and β-related bounding failures occur independently of each other.

$$P_{HMI,total} \le P_{HMI,FF+FM} + \binom{N}{1}\alpha\beta + \binom{N}{2}\alpha^2$$

Confidence level trade-off

The next step is to optimize the risk allocations for the three variables in the previous equation. In this context the following aspects have to be considered:

- The risk threshold of the user algorithm should be as high as possible in order to maximize user availability.
- α: The allocated overbounding failure risk per satellite should be increased as much as possible to enable SISA confidence level verification and to relax requirements on signal reliability. But anyway α must be small enough to keep the multiple-failure probability low.
- β: The maximum failure risk associated with the SISE bound should be just as small as necessary to minimize the impact of the single-failure case.

The optimal allocation depends on the application context. As basis of the simulations presented in this paper, the following allocations were chosen (all referring to 150s exposure interval):

User algorithm risk threshold	$P_{HMI,FF+FM}$ = 1.7e-7
Total SISA overbounding failure risk ($P_{fail,SISA}$)	α = 1.5e-5
SISE bound confidence level	β = 1.0e-4

Table 2: User integrity risk key parameters

Assuming N = 10, the following risk budget is obtained:

FF + FM, controlled by user algorithm	1.70e-7
Single failure, residual risk	1.50e-8
Multiple-failure case	1.01e-8
Margin	0.49e-8
TOTAL	2.00e-7

Table 3: User integrity risk budget

To give an example, the overbounding failure risk of 1.5e-5 can be further broken down to:

SISA confidence level to be verified: α_{verif} = 1.0e-5

Maximum probability of SIS failure: $P_{fail,Sat}$ = 0.5e-5

The main conclusion of this chapter is that by means of a user algorithm that covers both the fault-free and single-failure case it is possible to reach a user risk in the order of 1e-7 based on a SISE characterization (= SISA) that has only a confidence level in the order of 1e-5 to 1e-4. This is achieved by taking advantage of the multiplication of risk probabilities that are associated with the two complementary information sources SISA and SISE bound.

Impact on continuity risk

In Galileo the impact of a SISMA increase on continuity is very critical, because SISMA is very sensitive to monitoring station outages and generally behaves much more dynamically than SISA. Thanks to the different parameterization, such direct influence of the monitoring accuracy can be avoided in the SISE Bound concept.

According to its definition, the SISE bound is valid throughout the specified critical operation period, e.g. 15 s. Therefore, as long as no signal instability is indicated by a DU flag, it is justified that the user can keep using the initial SISE bound value until the end of the critical operation, even if an increased value is issued by the system during the operation. As a consequence, a short-term change of the monitoring accuracy has no direct impact on user continuity.

This approach is possible due to the fact that orbit and clock errors change only very slowly under nominal conditions. Signal jumps or large drifts occur with very low probability and can be detected by means of detection algorithms that process carrier phase observations, thereby triggering a DU flag. Note that jump and ramp detection on carrier phases is more sensitive to relative changes than the (absolute) SISE estimation. Finally, the maximum effect of small drifts that cannot be recognized with sufficient reliability should be covered by the additional SISE bound margin d_{cont}.

The small additional continuity risk that is caused by the jump and ramp detection has to be accounted for in the continuity fault tree.

USER ALGORITHM

This chapter is dedicated to a candidate user algorithm, which is compatible to the SISE bound integrity concept. It is largely analogous to the so-called Galileo adaptive user algorithm, which has been described in Oehler (2004b). Nevertheless a few important differences should be noted:

- The bias in the faulty mode (FM) cases is directly computed from the broadcast SISE bound instead of the reconstructed integrity check threshold.
- Since the uncertainty of the SISE estimation by the system is already included in the broadcast SISE bound's margin, the SISMA term is no longer necessary.
- The meaning of the FM occurrence probability is extended to include the probability that a sample is not covered by the verified SISA overbounding.

High-level algorithm logic

The algorithm is composed of the following computation steps:

- Satellite selection (optional)
- Position estimation
- Computation of topocentric estimator matrix
- Computation of fault-free case model parameters (FF)
- Computation of faulty mode model parameters (FM)
- Risk computation and threshold operation

The selection of satellites is not described here. It can be implemented, e.g., by means of a user-defined threshold or using iterative optimization. Detailed algorithms can be tailored to the specific application's need.

Position estimation

Based on the linear observation equations (N observations, 4 unknowns)

$$\underline{l} = \underline{H}\,\underline{x} + \underline{\varepsilon}$$

with

$$\underline{x} = (x_{ECEF}, y_{ECEF}, z_{ECEF}, \Delta t_{Rx})^T$$

and the observation covariance matrix

$$\underline{Q} = E\left\{\underline{\varepsilon}\underline{\varepsilon}^T\right\}$$

the least-squares position estimate is obtained by means of the linear position estimator matrix (dimension 4 x N).

$$\underline{x} = \underline{M}\,\underline{l}$$

$$\underline{M} = (\underline{H}^T \underline{Q}^{-1} \underline{H})^{-1} \underline{H}^T \underline{Q}^{-1}$$

Covariance propagation leads to the covariance matrix of estimated parameters (4 x 4):

$$\underline{\Sigma} = \underline{M}\ \underline{Q}\ \underline{M}^T = (\underline{H}^T \underline{Q}^{-1} \underline{H})^{-1}$$

Basic error propagation to topocentric coordinate system

For the computation of the covariance matrix in a local coordinate system (east, north, up) it is convenient to transform the position estimator matrix by means of a rotation matrix:

$$\underline{M}_{topo} = \underline{U}^T \left\{(\underline{H}^T \underline{Q}^{-1} \underline{H})^{-1} \underline{H}^T \underline{Q}^{-1}\right\}_{submatrix(3,N)}$$

$$\underline{U} = \left\{\underline{e}_{east}, \underline{e}_{north}, \underline{e}_{up}\right\}$$

This allows direct calculation of the topocentric position domain covariance matrix from the observation covariance matrix.

$$\underline{\Sigma}_{topo} = \underline{M}_{topo}\underline{Q}\ \underline{M}_{topo}^T$$

$$\underline{\Sigma}_{topo} = \begin{bmatrix} \sigma_e^2 & \sigma_{en} & \cdot \\ \sigma_{en} & \sigma_n^2 & \cdot \\ \cdot & \cdot & \sigma_u^2 \end{bmatrix}$$

This kind of covariance propagation is repeated for each FM(i) case with a different Q-matrix.

Fault-free case (FF) model parameters

In the FF case all pseudorange errors are assumed to follow *unbiased* (zero-mean) Gaussian distributions.

$$\varepsilon_{range(j),FF} \sim N\left(0, \sqrt{\sigma_{SISA,j}^2 + \sigma_{local,j}^2}\right)$$

Provided that the SISE of different satellites is mutually uncorrelated, the corresponding observation covariance matrix is diagonal. The variances refer to total range errors including SIS and local errors.

$$\left\{Q_{FF}\right\}_{j,j} = \sigma_{SISA,j}^2 + \sigma_{local,j}^2 \qquad \forall \ j = 1,\ldots,N$$

After mapping to the position domain, the resulting stochastic error model in the position domain is again unbiased Gaussian:

$$\underline{\varepsilon}_{topo,FF} \sim N(\underline{0}, \underline{\Sigma}_{topo,FF})$$

The horizontal error follows a general 2-dimensional Gaussian distribution. Since only the total length of the horizontal error is of interest, a conservative isotropic Gaussian model is used instead. The isotropic standard deviation is equal to the semi-major axis of the error ellipse that is equivalent to the square root of the larger eigenvalue of the corresponding horizontal covariance matrix.

$$\xi = \sqrt{\frac{\sigma_e^2 + \sigma_n^2}{2} + \sqrt{\left(\frac{\sigma_e^2 - \sigma_n^2}{2}\right)^2 + \sigma_{en}^2}}$$

Faulty mode (FM) model parameters

The faulty mode computation considers all possible single-failure cases. Consequently the computation has to be repeated for each sub-case FM(i) with faulty or not overbounded signal at satellite i.

In the faulty mode FM(i) all signals are characterized as in the FF scenario except signal i, which is assumed to have a deterministic bias equal to the broadcast SISE bound (B). Note that the biased signal is nevertheless affected by local measurement errors.

$$\varepsilon_{range(j),FM(i)} \sim \begin{cases} N(B_j, \sigma_{local,j}) & \text{for } j = i \\ N\left(0, \sqrt{\sigma_{SISA,j}^2 + \sigma_{local,j}^2}\right) & \text{for } j \neq i \end{cases}$$

Accordingly only one diagonal element of the Q-matrix differs from the FF case:

$$\left\{Q_{FM(i)}\right\}_{j,j} = \begin{cases} \sigma_{local,j}^2 & \text{for } j = i \\ \sigma_{SISA,j}^2 + \sigma_{local,j}^2 & \text{for } j \neq i \end{cases}$$

The resulting error model in the position domain is a biased Gaussian distribution.

$$\underline{\varepsilon}_{topo,FM(i)} \sim N(\underline{\mu}_{topo,FM(i)}, \underline{\Sigma}_{topo,FM(i)})$$

The components of the bias vector can be obtained by linear mapping of the signal bias using the topocentric estimator matrix.

$$\underline{\mu}_{topo,FM(i)} = \left[\mu_{e,FM(i)} \quad \mu_{n,FM(i)} \quad \mu_{u,FM(i)}\right]^T$$

$$\mu_{e,FM(i)} = \left| M_{topo}[1,i] \; B_i \right|$$

$$\mu_{n,FM(i)} = \left| M_{topo}[2,i] \; B_i \right|$$

$$\mu_{u,FM(i)} = \left| M_{topo}[3,i] \; B_i \right|$$

For the isotropic horizontal error model again only the length of the horizontal bias vector is relevant.

$$\mu_{hor,FM(i)} = \sqrt{\mu_{e,FM(i)}^2 + \mu_{n,FM(i)}^2}$$

Integrity risk

Finally the user integrity risk can be computed from the stochastic model parameters for FF and FM case and the failure risk $P_{fail,SISA}$. Recall that $P_{fail,SISA}$ is supposed to cover both SIS failure and SISA overbounding failure risk. The following relations are obtained by integration of the respective probability density functions.

The total integrity risk is computed as sum of horizontal and vertical integrity risk. This is a conservative assumption, since the probability spaces of a horizontal HMI and a vertical HMI, respectively, are overlapping.

Vertical integrity risk at vertical alert limit (VAL):

$$P_{HMI,vert}(VAL) = 1 - erf\left(\frac{VAL}{\sqrt{2}\;\sigma_{u,FF}}\right) + \sum_{i=1}^{N}\left\{P_{fail,SISA(i)}\left[1 - cdf_N\left(\frac{VAL}{\sigma_{u,FM(i)}}, \frac{\mu_{u,FM(i)}}{\sigma_{u,FM(i)}}\right)\right]\right\}$$

with the error function defined as

$$erf(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-\frac{t^2}{2}}dt$$

and the cumulative distribution function of a non-central Gaussian distribution (argument x, normalized mean λ):

$$cdf_N(x,\lambda) = \tfrac{1}{2}erf(x+\lambda) + \tfrac{1}{2}erf(x-\lambda)$$

Horizontal integrity risk at horizontal alert limit (HAL):

$$P_{HMI,hor}(HAL) = \exp\left(-\frac{HAL^2}{2\xi_{FF}^2}\right) + \sum_{i=1}^{N}\left\{P_{fail,SISA(i)}\left[1 - cdf_{\chi^2}\left(\frac{HAL^2}{\xi_{FM(i)}^2}, \frac{\mu_{hor,FM(i)}^2}{\xi_{FM(i)}^2}\right)\right]\right\}$$

with the cumulative distribution function of a non-central χ^2- distribution with degree of freedom 2 (argument x, non-centrality parameter λ):

$$cdf_{\chi^2}(x,\lambda) = \int_0^x pdf_{\chi^2}(t,\lambda)dt$$

$$pdf_{\chi^2}(x,\lambda) = \tfrac{1}{2}e^{-\frac{1}{2}(x+\lambda)} \cdot \sum_{i=0}^{\infty}\frac{x^i\lambda^i}{2^{2i}(i!)^2}$$

The total integrity risk is obtained from the relation

$$P_{HMI}(HAL,VAL) = P_{HMI,hor}(HAL) + P_{HMI,vert}(VAL)$$

Availability of integrity is given, whenever the computed risk is below the value that is allocated to the user equation.

$$P_{HMI}\begin{cases} \le P^{max}_{HMI,FF+FM} \Rightarrow \text{available} \\ > P^{max}_{HMI,FF+FM} \Rightarrow \text{not available} \end{cases}$$

SIMULATION RESULTS

Computer simulations have been carried out to assess the availability performance of the modified user algorithm (using the SISE bound parameter) with respect to the Galileo baseline algorithm (using SISMA). Unlike the simulations reported in Mach (2006) the new simulations include the actual processing of the integrity processing facility. For this purpose satellite orbit and clock errors as well as monitoring observation errors have been simulated using artificial Gaussian random errors.

Risk probability vs. protection levels

Availability performance is usually expressed in terms of probability. Such results, however, always refer to specific alert limit values and measurement accuracy assumptions. To give an example, for appropriately large alert limits any algorithm will yield 100 % availability, whereas for other specific values the difference between two algorithms may be misleadingly large. The results of the performed simulations will therefore be reported hereafter in terms of protection levels (PL), which represent a much more intuitive, metric measure for the purpose of comparison.

The computation of PL values requires risk allocations for the vertical and horizontal HMI risk. For our simulations it was decided to use for both VPL and HPL the total maximum risk. This refers to users that are interested only in the vertical or horizontal position, respectively. Even though this is not precise for the general case, it nevertheless provides a good basis for the relative comparison of the two algorithms.

There is no direct formula for the computation of PL values, but they can be easily determined by searching iteratively those HAL/VAL values, for which the risk is equal or close to the allocated risk. Usually less than 10 iterations are necessary to achieve a precision of 5 cm.

Simulation assumptions

The key parameters of the two algorithms were chosen as follows:

- Galileo user algorithm:
  SISA = 0.85 m; α = 1.5e-5;
  SISMA based on simulated monitoring data
- SISE Bound algorithm:
  α = 1.5e-5, β = 1e-4 (k_B = 3.89), d_{cont} = 0.3 m;
  estimated SISE based on simulated monitoring data;

The monitoring observations were simulated based on the following assumptions:

- Simulated SISE: Gaussian random with following standard deviations (equivalent to SISE of 0.70 m)
  orbit errors: 0.25 m (radial), 1.25 m (across-track, along-track); satellite clock error: 1.67 ns
- Simulated range observation errors: Gaussian random with exponential elevation dependency:
  90 deg: 0.30 m; 10 deg: 0.80 m (std. dev.)

For both algorithms the maximum allowed risk that defines the protection levels was set to 1.7e-7/150 s, for HPL as well as for VPL computations. For each algorithm the following three scenarios were considered:

SCEN	DESCRIPTION
27_Nom	Nominal Galileo constellation (Walker 27/3/1, 27 satellites) and monitoring network (40 stations)
26_Nom	1 satellite failure (26 satellites), nominal network
26_Dgr	1 satellite failure + 1 station failure (degraded station network with 39 stations)

Table 4: Simulation scenarios

All computations were based on an equi-angular user grid with 5 degrees spacing and an assumed elevation cut-off angle of 10 degrees (for both user and monitoring stations). Simulations were run over one constellation repeat cycle (10 days) with a sampling interval of 120 s.

Results & comparison

The simulation results are illustrated by means of two-dimensional VPL/HPL histograms for both Galileo and SISE Bound algorithm (see figures on next page). The histograms for the nominal constellation scenario (Figure 4) already show that, although maximum values are similar, the occurrence rate of large PL values is slightly smaller for the SISE Bound algorithm. For the “1 satellite failure” scenario (Figure 5) this effect is quite significant.

When looking at the results of the “1 satellite + 1 station failure” scenario (Figure 6), one can see another difference: While Galileo protection levels are clearly deteriorated for the degraded network, the SISE bound results are almost unaffected. The reason for this is the strong sensitivity of SISMA to station outages, caused by the rule that it always has to cover the worst additional failure. As explained earlier in the remarks on continuity, this is not necessary for the SISE Bound concept.

SISE Bound *GALILEO*

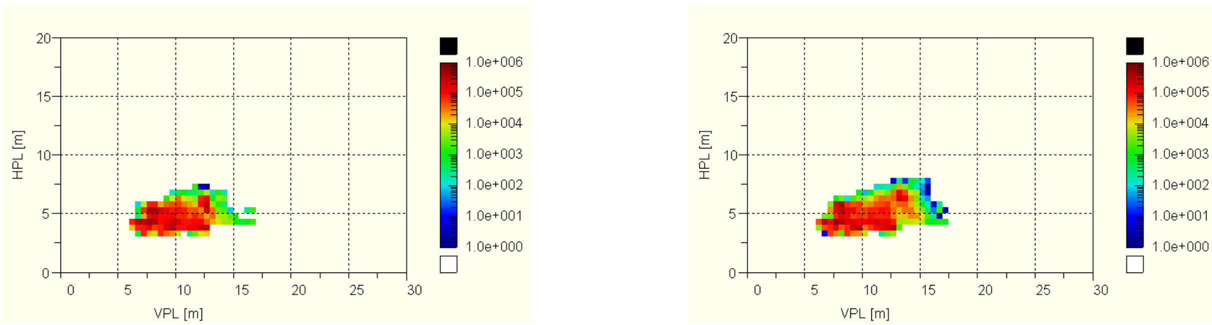

Figure 4: Protection level histograms for nominal Galileo constellation and station network (scenario 27_Nom); performance comparison of SISE Bound (left) and GALILEO (right)

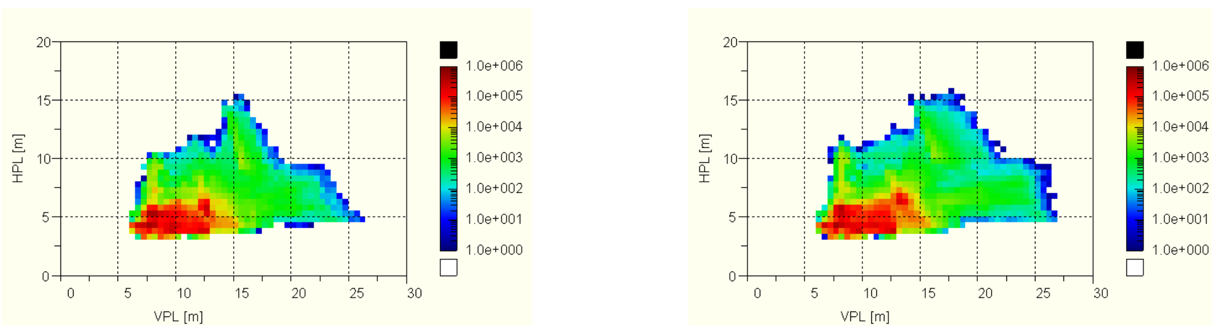

Figure 5: Protection level histograms for 1 satellite failure with nominal network (scenario 26_Nom); performance comparison of SISE Bound (left) and GALILEO (right)

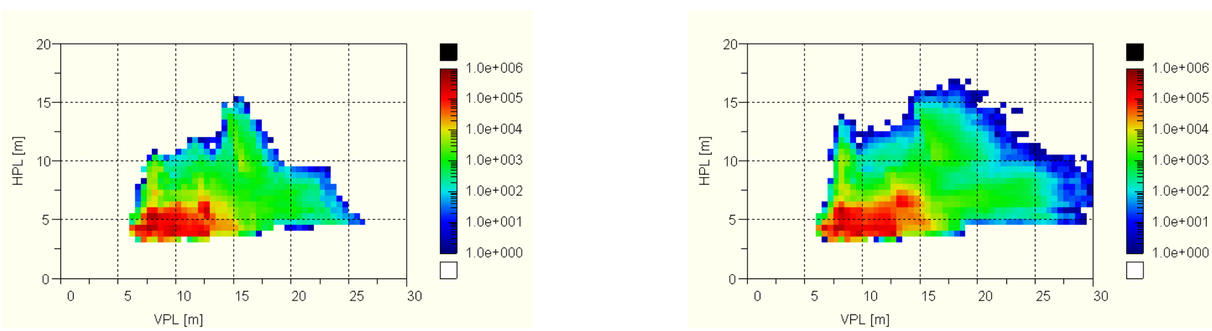

Figure 6: Protection level histograms for 1 satellite failure + 1 station failure (scenario 26_Dgr); performance comparison of SISE Bound (left) and GALILEO (right)

ALGO	SCEN	MEAN	SDEV	90 %	99 %	99.9 %
SISE-B	27_Nom	9.6	2.0	12.6	13.3	14.8
GAL	27_Nom	10.0	2.1	13.1	14.5	15.5
SISE-B	26_Nom	10.1	2.4	12.8	16.7	21.5
GAL	26_Nom	10.5	2.6	13.6	18.1	23.8
SISE-B	26_Dgr	10.1	2.4	12.8	16.7	21.6
GAL	26_Dgr	10.7	2.7	13.9	18.5	24.7

Table 5: VPL statistics (all values in meters)

The VPL statistics table (Table 5) confirms the impression given by the diagrams. In terms of mean values, it shows an average improvement by the new algorithm in the order of 5 %. More interesting is the comparison of percentiles referring to high availabilities. For the 99.9 % percentile the SISE Bound algorithm outperforms the Galileo user algorithm by more than 3 m or 13 % (for scenario 26_Dgr). It can be concluded that the new concept reduces in particular the probability of very large values, which lead to non-availability. This is exactly what is needed for an integrity system with high-performance availability requirements.

The main reason for the better performance is the fact that the SISE bound more closely limits the maximum bias that is assumed in the faulty-mode terms of the user equation. In Figure 7 an exemplary histogram of SISE bound values is depicted. For the considered scenario most samples are scattered around a mean value of about 2.5 m. The corresponding bias values in the Galileo user equation (threshold TH) typically amount to 5-6 m and do not yet contain the monitoring uncertainty (which is already covered by the SISE bound)! In other words, the SISE bound contains more useful information for the user than the SISMA parameter.

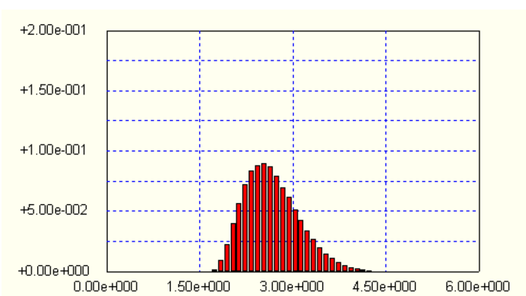

Figure 7: Histogram of SISE bound values (scenario 27_Nom)

As a matter of fact, the exact impact of the new concept in terms of PL or availability depends on several parameters as for example the faulty mode probability. Nevertheless the presented results seem to indicate a moderate but significant improvement in the order of 5-10 %.

CONCLUSIONS

System engineering aspects

The simulation results confirm our expectation that the SISE bound approach is more efficient in terms of user availability. This can be exploited to either enhance performance or alternatively to relax critical system requirements.

Apart from this increased efficiency, the new concept offers further advantages for the system:

- First of all, the described user protection approach establishes a mathematical justification to relax SISA overbounding requirements and still ensure user integrity risks in the order of 10^{-7}.
- Regarding system verification there is a twofold advantage. Firstly, the SISA overbounding has to be verified only up to a well-defined (though very high) confidence level. Secondly, the SISE bound's reliability has to be tested only for one distinct confidence level. SISMA is no longer a user interface parameter, for which overbounding would have to be proven.
- Another aspect is that the concept of the integrity check threshold is dropped. Thereby also the fixed formula for the computation of the threshold, as it is foreseen for Galileo, becomes obsolete. Such a fixed formula represents a strong constraint for the system itself that is likely to hamper internal system upgrading and tuning.
- Unlike the conventional integrity flag, the SISE bound provides quantitative information about how good the signal actually is. The user interface gives all freedom to develop appropriate satellite selection algorithms, tailored to the specific application. Under certain conditions this allows to optimise even accuracy performance, not only availability.

Conceptual aspects

From a conceptual point of view, the new approach is characterized first of all by a clear and intuitive definition of the meaning of the integrity message. The SISE bound is a metric measure that everyone can immediately understand. Apart from the obvious fact that integrity data always refers to the applicable navigation data, it does not depend on any other data but represents a self-contained statement. (This is not the case in the Galileo baseline, where the integrity flag refers to a threshold that depends on SISA.)

The SISE bound parameter represents a truly instantaneous parameter. This corresponds exactly to the complementary role of the orbit determination & clock synchronization on the one hand and the integrity determination on the other. While the navigation message contains long-term ephemeris, clock and statistical (SISA) data, the integrity message provides an instantaneous characterization of the actual signal state.

Thanks to this clear separation, the SISE bound data can be both generated and verified independently of any internal system parameters. Besides the performance of SISE bound data can be monitored and assessed in a straightforward manner by an external authority. This makes it a promising candidate for augmentation systems and external regional integrity systems (ERIS) too.

REFERENCES

DeCleene, B. (2000); *Defining Pseudorange Integrity-Overbounding*, Proceedings of the ION GPS 2000, Salt Lake City, Utah, 19-22 September 2000, pp. 1916-1924

Herraiz-Monseco, E., et al. (2001); *A New System Level Integrity Concept for Galileo: The Signal in Space Accuracy*, Proceedings of the ION GPS 2001, Salt Lake City, Utah, 11-14 September 2001, pp. 1304-1316

Mach, J., Werner, W. (2006); *An alternative Integrity Concept for GNSS based on Signal-in-Space Error Bounds*, Proceedings of the European Navigation Conference 2006, Manchester, United Kingdom, 07-10 May 2006

Oehler, V., et al. (2004); *The Galileo Integrity Concept*, Proceedings of ION GNSS 2004, Long Beach, California, 21-24 September 2004, pp. 604-615

Oehler, V., et al. (2004b); *User Integrity Risk Computation at the Alert Limit without Fixed Allocations*, Proceedings of the ION GNSS 2004, Long Beach, California, 21-24 September 2004, pp. 1645-1652

Oehler, V., et al. (2005); *The Galileo Integrity Concept and Performance*, Proceedings of GNSS 2005 – The European Navigation Conference, Munich, Germany, 19-22 July 2005

RTCA (2001); *Minimum Operational Performance Standards For Global Positioning System / Wide Area Augmentation System Airborne Equipment*, RTCA Document No.: RTCA/DO-229C, RTCA Inc., Washington D.C., 28 November 2001